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IN pp COLLISIONS

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A Unified Description of  
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ABSTRACT

We show that over a very wide range of center of mass angles, energies, and transverse momenta, the invariant cross sections for inclusive production of hadrons in pp collisions show a very simple behavior when expressed in terms of a new radial scaling variable

$$x_R \equiv p^* / p_{\max}^*$$

In a separate letter,<sup>1</sup> we have presented data on inclusive  $\pi^0$  production from pp collisions at NAL, showing that for center of mass (c.m.) angles from  $40^\circ$  to  $110^\circ$ , transverse momenta from 0.3 to 4.3 GeV/c, and incident energies from 50 to 400 GeV, the invariant cross sections factorize very simply into the product of two universal functions

$$E \frac{d^3\sigma}{dp^3} = f(x_R) g(p_\perp) \quad (1)$$

where  $x_R$  is an angle-independent scaling variable

$$x_R \equiv p^* / p_{\max}^* \approx 2p^* / \sqrt{s}. \quad (2)$$

The observed dependence on  $x_R$  can be approximated by

$$f(x_R) \propto (1 - x_R)^4 \quad (3)$$

and the intrinsic transverse momentum dependence by

$$g(p_\perp) \propto (p_\perp^2 + m^2)^{-4.5}, \quad m^2 = 0.86 \text{ GeV}^2. \quad (4)$$

The simplicity of this factorization and the wide kinematic range over which we have found it applicable make it desirable to examine other pp inclusive data. Although the available data are limited, we feel it is important to provide an initial overview of an emerging pattern for inclusive measurements in strong interactions.

We have compared our results with other inclusive pion measurements near  $90^\circ$  in the c.m., by the Chicago-Princeton group at NAL<sup>2</sup> ( $\pi^\pm$ ) and the CCR collaboration at the ISR<sup>3</sup> ( $\pi^0$ ). In the region of energy and

transverse momentum where the data overlap, the agreement is satisfactory. However, we would like to test the scaling behavior over as wide a range as possible of center of mass angles and energies, beyond the region covered by our experiment. For this, we consider recent data from the CERN-Bologna group<sup>4</sup> at the ISR at smaller c. m. angles ( $5^\circ$ - $20^\circ$ ) and higher energies, and lower-energy data from Allaby et al.<sup>5</sup> ( $P_{inc} = 24$  GeV/c) and Akerlof et al.<sup>6</sup> ( $P_{inc} = 12.4$  GeV/c). To minimize diffractive or leading-particle effects, we restrict ourselves to negative produced particles. Figure 1 shows the  $x_R$  dependence, for fixed  $p_\perp$ , of invariant cross sections for  $p + p \rightarrow \pi^- + (\text{anything})$ . The open circles represent the ISR data at  $p_\perp = 0.8$  GeV/c and show a dependence on  $x_R$  in good agreement with that found in our  $\pi^0$  results, shown as the solid curve (arbitrarily normalized to pass through the points at small  $x_R$ ). This agreement is rather remarkable considering the different kinematic regions covered by the two sets of data. We have extended the comparison to include the lower-energy data from Allaby et al. at  $p_\perp = 0.8$  GeV/c and from Akerlof et al. at  $p_\perp = 1.0$  GeV/c, which cover a wide range of c. m. angles and extend rather close to the kinematic boundary  $x_R = 1$  (see insert in Fig. 1). The results of Akerlof et al., when scaled from  $p_\perp = 1.0$  GeV/c to  $p_\perp = 0.8$  GeV/c using Eq. (4), agree very well in absolute normalization with the data of Allaby et al. at  $p_\perp = 0.8$  GeV/c. However, both must be scaled down by about 40% to line up with the ISR points in the region of  $x_R$  where there is overlap (this could be due to a slight

deviation from scaling over the enormous energy spread involved, or to experimental normalization differences). When this is done, as shown in Fig. 1, it is apparent that the  $x_R$  dependence is very similar for all four sets of data. It therefore appears that a single universal scaling expression is consistent with inclusive pion data from pp collisions, independent of c.m. angle from  $5^\circ$  to  $110^\circ$  and for  $25 \lesssim s \lesssim 4000$ , if the data are expressed in terms of the variable  $x_R$ .

Thus encouraged, we are led to ask whether a similar universality exists for  $K^-$  and  $\bar{p}$  data. The Chicago-Princeton<sup>2</sup> results taken at NAL near  $90^\circ$  in the c.m. can be compared with measurements taken at very small c.m. angles by the CERN-Bologna group.<sup>4</sup> Since in the NAL data both  $p_\perp$  and  $x_R$  are varying, we assume that the intrinsic  $p_\perp$  dependences  $g(p_\perp)$  for  $\pi^-$ ,  $K^-$ , and  $\bar{p}$  are the same for large enough  $p_\perp$ . We then divide out the  $p_\perp$  dependence in the NAL data by comparing particle ratios  $K^-/\pi^-$  and  $\bar{p}/\pi^-$  between the two experiments. This comparison is shown in Fig. 2. The upper set of points in Fig. 2a show the  $K^-/\pi^-$  ratios measured by the Chicago-Princeton experiment for  $p_\perp > 3$  GeV/c. The ratio of  $K^-/\pi^-$  can be seen to fall with increasing  $x_R$ . This means, given the assumption of similar forms of  $g(p_\perp)$  for  $K^-$  and  $\pi^-$ , that the  $K^-$  radial scaling function has a more rapid  $x_R$  dependence than that for the  $\pi^-$  mesons. The CERN-Bologna data were taken at a fixed  $p_\perp$  of 0.8 GeV/c and are shown as the lower set of points in Fig. 2a. Although the errors on the data are large it appears that the ratio of  $K^-/\pi^-$  again decreases

with increasing  $x_R$ . In fact a comparison of the two sets of data, one taken at  $90^\circ$  and the other at very small angles in the center of mass, shows that they are consistent with being parallel and therefore with having the same  $K^-$  radial scaling function  $f_{K^-}(x_R)$ . The magnitudes of the ratios  $K^-/\pi^-$  from the two experiments are different, but this difference may arise from the low  $p_\perp$  value of the ISR data. A simple mechanism for describing this difference is to assume the form of Eq. (4) with a species-dependent mass term. To illustrate this, curves are shown in Fig. 2a which are a good fit to the observed  $K^-/\pi^-$  ratios. The functions used to generate the curves are

$$E \frac{d^3\sigma}{dp^3} (\pi^-) = N(p_\perp^2 + 0.86)^{-4.5} (1 - x_R)^4 \quad (5)$$

and

$$E \frac{d^3\sigma}{dp^3} (K^-) = 0.36 N(p_\perp^2 + 1.22)^{-4.5} (1 - x_R)^5 \quad (6)$$

where  $N$  is an overall normalization constant. Thus a species-dependence in the mass term in  $g(p_\perp)$  can significantly affect the magnitude of the particle ratio at small transverse momentum.

Figure 2b shows the same comparison for the  $\bar{p}/\pi^-$  ratios measured at  $90^\circ$  by the Chicago-Princeton group at NAL and at small angles by the CERN-Bologna group at the ISR. In this case the data from the two experiments allow a more detailed comparison. The ratio of  $\bar{p}/\pi^-$  at  $90^\circ$  can be seen to fall rapidly with increasing  $x_R$ . This means, given the assumption

of similar forms of  $g(p_{\perp})$  for  $\bar{p}$  and  $\pi^{-}$  that the  $\bar{p}$  radial scaling function has a much more rapid  $x_R$  dependence than that for  $\pi^{-}$  mesons. The small angle ISR  $\bar{p}/\pi^{-}$  data are strikingly parallel to the large angle NAL data. Thus both sets of data are consistent with having the same  $\bar{p}$  radial scaling function  $f_{\bar{p}}(x_R)$ .

Once again the magnitudes of the ratios  $\bar{p}/\pi^{-}$  from the two experiments are different. As before we ascribe this to a species dependence in the mass term of  $g(p_{\perp})$  in Eq. (4). The curves shown in Fig. 2b were generated from Eq. (5) for the  $\pi^{-}$  cross section and the following expression for  $\bar{p}$  production:

$$E \frac{d^3\sigma}{dp^3} (\bar{p}) = 0.26 N(p_{\perp}^2 + 1.04)^{-4.5} (1 - x_R)^7. \quad (7)$$

In conclusion, we have shown that when the invariant cross sections are expressed in terms of a new scaling variable  $x_R$ , available data from the ZGS, CERN PS, NAL and the ISR on inclusive  $\pi^0$ ,  $\pi^{-}$ ,  $K^{-}$ , and  $\bar{p}$  production are consistent with

a) a factorized form:

$$E \frac{d^3\sigma}{dp^3} (\pi) = f_{\pi}(x_R) g_{\pi}(p_{\perp})$$

$$E \frac{d^3\sigma}{dp^3} (K^{-}) = f_{K^{-}}(x_R) g_{K^{-}}(p_{\perp})$$

$$E \frac{d^3\sigma}{dp^3} (\bar{p}) = f_{\bar{p}}(x_R) g_{\bar{p}}(p_{\perp}),$$

where

- b) the intrinsic  $p_{\perp}$  dependences  $g(p_{\perp})$  for  $\pi^0$ ,  $K^-$ , and  $\bar{p}$  are the same for large enough  $p_{\perp}$ , i. e.

$$g_{\pi}(p_{\perp}) \approx g_{K^-}(p_{\perp}) \approx g_{\bar{p}}(p_{\perp}), \quad p_{\perp} \gg m_i,$$

and

- c) the radial scaling functions are given approximately by

$$f_{\pi}(x_R) = (1 - x_R)^4$$

$$f_{K^-}(x_R) = (1 - x_R)^5$$

$$f_{\bar{p}}(x_R) = (1 - x_R)^7.$$

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data from this experiment agree with our results where the energy  
ranges overlap, at higher energies their cross sections vary more  
rapidly with  $x_R$  for fixed  $p_{\perp}$ . This could be due to an energy-dependent  
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FIGURE CAPTIONS

- Fig. 1 Invariant cross sections for  $p + p \rightarrow \pi^- + (\text{anything})$  are plotted versus  $x_R$  for fixed  $p_{\perp}$ . The solid curve represents the  $x_R$  dependence found in our experiment. Arbitrary normalization shifts have been made to illustrate the agreement in the  $x_R$  dependence. The inset shows the values of  $x_{\perp} \equiv p_{\perp} / p_{\perp \text{max}}^*$  and  $x_{\parallel} \equiv p_{\parallel}^* / p_{\parallel \text{max}}^*$  covered by the three sets of data points.
- Fig. 2 Particle ratios  $K^- / \pi^-$  and  $\bar{p} / \pi^-$  are plotted versus  $x_R$  for data from Ref. 2 and Ref. 4. The curves were generated by Eqs. (5), (6), and (7).

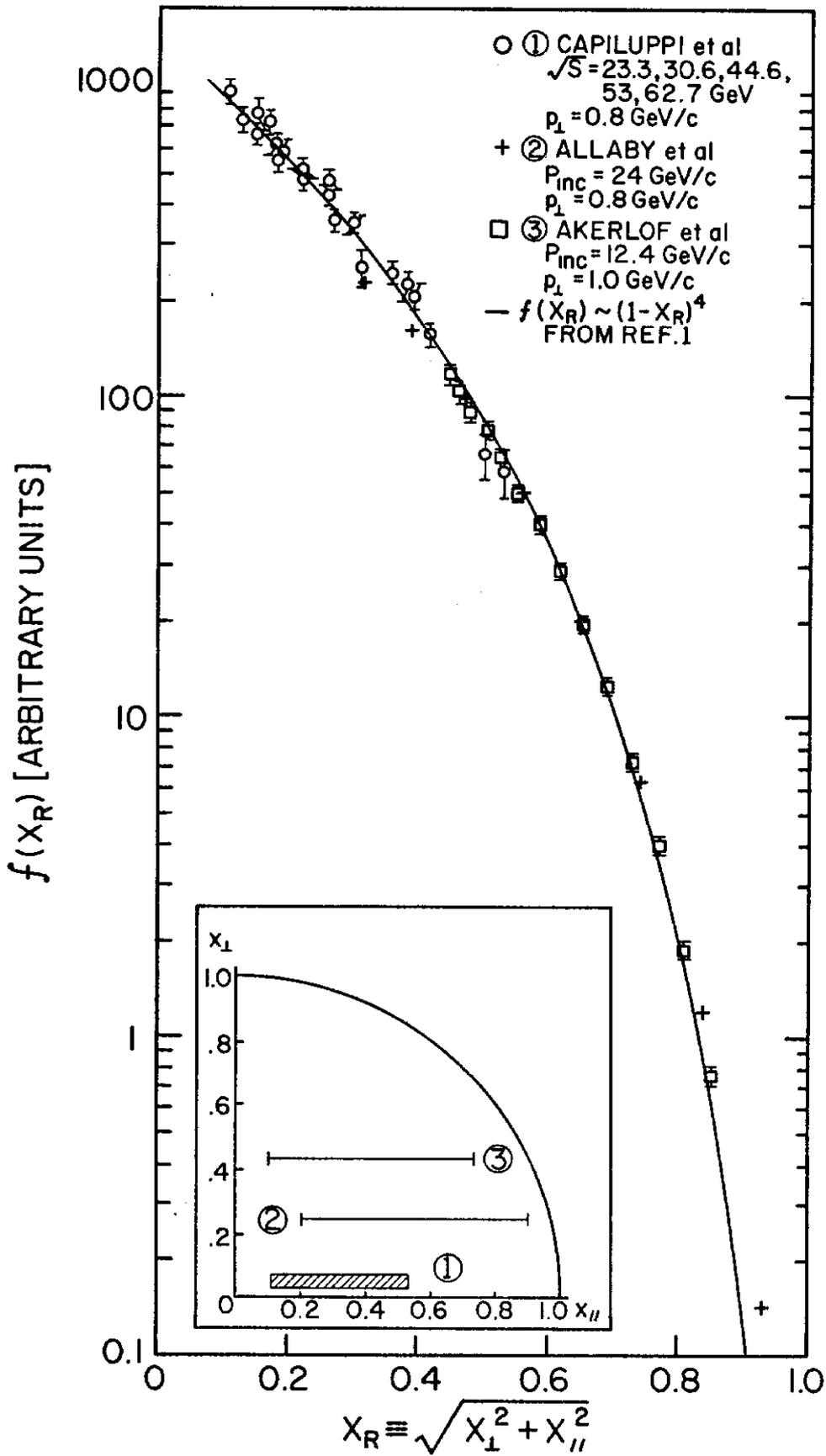


Fig. 1

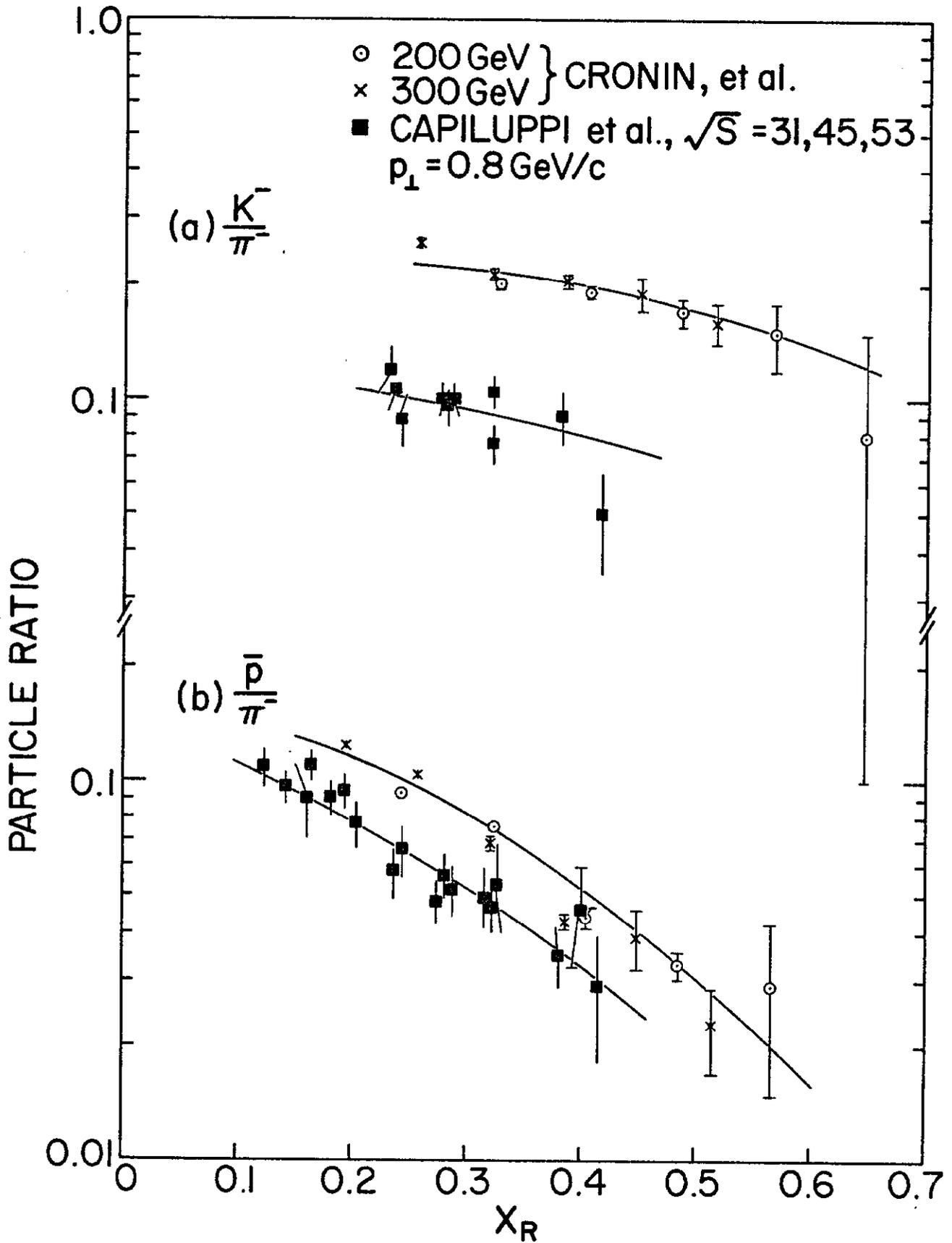


Fig. 2