



IMPACT PICTURE PREDICTIONS FOR  $K^+p$  SCATTERING AT NAL

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ABSTRACT

On the basis of the Impact Picture we show that at NAL energies there will be rather little shrinkage of the forward  $K^+p$  elastic peak. This is in distinct contrast to the experimental results for  $pp$  scattering. In addition, the total  $K^+p$  elastic cross section will increase faster than the  $K^+p$  total cross section with increasing energy. Specifically, the integrated elastic cross section is estimated to increase by about 12% in the NAL energy range.

In 1970, on the basis of relativistic quantum field theory, we described the general qualitative features of the Impact Picture of high energy scattering.<sup>1</sup> Subsequently in 1972, we made quantitative predictions for experiments at the CERN Intersecting Storage Rings.<sup>2</sup> At that time we predicted:

- (i) the proton-proton total cross section would increase by about 3 mb. Results of experiments<sup>3</sup> in 1973 showed a significant rise in the proton-proton total cross section (In fact the rise was about 5 mb).
- (ii) a dip would develop in the elastic proton-proton differential cross section at  $t \approx 1.3$ . This also was subsequently confirmed.<sup>4</sup>

We have used the observed rise of 5 mb in the proton-proton total cross section to determine the value of the parameter  $c$ <sup>5</sup>. As the same parameter  $c$  controls all the rates of increase of hadronic total cross sections, we then made predictions for the energy dependence of  $\pi^+$ ,  $K^+$  and  $\bar{p}$  total cross sections at high energy.

Very recently, preliminary measurements<sup>6</sup> at the National Accelerator Laboratory have been shown to be in excellent agreement with all of these predictions. Thus encouraged we are led to consider additional consequences of the Impact Picture which could be tested at NAL in the near future.

In this paper we discuss two such consequences for the  $K^+p$  system. First, we predict rather little shrinkage of the  $K^+p$  forward elastic peak. Secondly, we predict that the integrated

elastic  $K^+p$  cross section should rise by about 12% in the energy range  $(100 \leq s(\text{GeV})^2 \leq 400)$  available at the National Accelerator Laboratory.

In the energy range of NAL, the dominant Regge backgrounds of  $f^0$  and  $\omega$  exchanges cannot be neglected a priori. We shall therefore include the contributions of  $f^0$  and  $\omega$  in a phenomenological manner in our model of the Impact Picture. Since there are uncertainties in the detailed forms of these Regge background terms, the predictions here are meaningful only if they are not sensitive to these uncertainties. This is indeed the case for  $K^+p$  scattering. The elastic kaon-proton scattering amplitude thus consists of three terms, namely

1. the pomeron as described and parameterized by us previously;<sup>5</sup>
2. tensor exchanges ( $f^0, A_2$ ) with even signature and trajectory  $\alpha(t) = 0.5 + t$ ,  $t$  in  $(\text{GeV}/c)^2$ ; and
3. vector exchanges ( $\rho, \omega$ ) with odd signature and trajectory degenerate with  $\alpha(t)$ .

We have neglected the spin-flip amplitude, which is zero in the exactly forward direction, and isospin-exchange contributions, which are known to be small from measurements on charge-exchange scattering.<sup>7</sup>

The  $f^0$  and  $\omega$  type exchanges are assumed in the usual way to give rise to amplitudes of the form

$$A_{f^0}(s, t) = \gamma_f(t) \left[ \frac{\alpha(t)}{\alpha(0)} \right] \left\{ \frac{1 + \exp[-i\pi\alpha(t)]}{\sin \pi \alpha(t)} \right\}_s [\alpha(t) - 1]$$

and

$$A_\omega(s, t) = \gamma_\omega(t) \left\{ \frac{1 - \exp[-i\pi\alpha(t)]}{\sin \pi \alpha(t)} \right\}_s [\alpha(t) - 1]$$

where  $\gamma_{f,\omega}(t)$  is the residue function,  $s$  is in units of  $\text{GeV}^2$ .

The differential cross sections are given by

$$\frac{d\sigma}{dt}(K^+p) = 1.2233 \left| \frac{1}{s} A_p + A_{f^0} - A_\omega \right|^2 \text{ mb}/(\text{GeV}/c)^2$$

and

$$\frac{d\sigma}{dt}(K^-p) = 1.2233 \left| \frac{1}{s} A_p + A_{f^0} + A_\omega \right|^2 \text{ mb}/(\text{GeV}/c)^2.$$

where  $A_p$  is the previously defined pomeron amplitude.

Fits by other authors<sup>8</sup> to  $pp$  and  $\bar{p}p$  elastic scattering at less than 20 GeV have led to the parameterization

$$\gamma_\omega(t) = \bar{\gamma}_\omega \exp(0.5 t) (1 + t/0.14) [(1 + t)^2 + 0.16]/1.16.$$

This parameterization reproduces the well-known crossover effect in the elastic differential cross section at  $|t| = 0.14 (\text{GeV}/c)^2$ . Since  $K^+p$  and  $K^-p$  have the same crossover effect at approximately<sup>9</sup> this value of  $|t|$ , we take the same residue function and apply it to the  $Kp$  system with no new parameters.

The residue  $\gamma_f(t)$  has not previously been determined without some theoretical assumption on the pomeron contribution.

For simplicity we take

$$\gamma_f(t) = \bar{\gamma}_f \exp(dt)$$

where  $d$  is a single adjustable parameter.

From the optical theorem we have

$$\sigma_{\text{total}}(K^-p) = 4.893 \left[ \frac{\text{Im } A_p(s,0)}{s} + s^{-1/2} (\bar{\gamma}_f + \bar{\gamma}_\omega) \right] \text{mb}$$

and

$$\sigma_{\text{total}}(K^+p) = 4.893 \left[ \frac{\text{Im } A_p(s,0)}{s} + s^{-1/2} (\bar{\gamma}_f - \bar{\gamma}_\omega) \right] \text{mb}$$

We have previously<sup>5</sup> obtained the following parameters for the case of kaon-nucleon scattering:

$$c = .082925, c' = 0, \lambda = .60071, x_0(Kp) = 1.4295, f(Kp) = 1.3452$$

$$4.893 (\bar{\gamma}_f + \bar{\gamma}_\omega) = 29.38 \text{ and } 4.893 (\bar{\gamma}_f - \bar{\gamma}_\omega) = 8.25$$

Hence the residue normalizations are given by  $\bar{\gamma}_f = 3.85$  and  $\bar{\gamma}_\omega = 2.16$ .

Thus as stated earlier we are left with one free parameter, namely  $d$ , which can be adjusted for a satisfactory description of the slope parameter of  $K^\pm p$  elastic scattering. We find that a reasonable fit to the data is obtained with  $d = 0.5 \text{ (GeV/c)}^{-2}$ .

In Fig. 1 we show with solid lines our results on the slope parameter  $b$ , defined as the logarithmic derivative of  $\frac{d\sigma}{dt}$ . The dashed lines give the corresponding curves when only the pomeron term is included. Available data<sup>10</sup> on the slope parameter are also shown for comparison. For  $K^+p$ , the difference between the

solid and dashed curves is very small. Due to the uncertainty in the form of the Regge contributions from  $f^0$  and  $\omega$ , we therefore prefer to restrict our predictions to the  $K^+p$  channel and for sufficiently high laboratory energies, say greater than 50 GeV.

From Fig. 1 we see that there is very little shrinkage in this  $|t|$  range for the  $K^+p$  diffraction peak above 50 GeV. At small  $|t|$ , say  $|t| \approx 0.06$ , there is a slight amount of shrinkage and  $\alpha'_{\text{eff}} = 1/2 \frac{db}{d(\ln s)} \approx 0.1 \text{ GeV}^{-2}$  at 100 GeV kaon laboratory energy. This is in distinct contrast to the experimental results<sup>11</sup> for proton-proton scattering, where the corresponding values of  $\alpha'_{\text{eff}}$  are found to be in the range 0.28 to 0.47  $\text{GeV}^{-2}$ .

Figure 2 shows  $\sigma_{\text{el}}$ , the integrated elastic cross section versus  $s$ . We see that there is a rapid rise of  $\sigma_{\text{el}}$  ( $s$ ) taking place beyond 50 GeV incident laboratory energy. This rate of increase of the  $K^+p$  elastic cross section is greater than that of the  $K^+p$  total cross section given previously.<sup>5</sup> In fact, the predicted rise in  $\sigma_{\text{el}}$  is about 12% in the range  $100 < s(\text{GeV})^2 < 400$ .

In Fig. 3 we show the predicted differential cross section for  $K^+p$  elastic scattering. The curve includes the contribution of the pomeron term only. This contribution changes very little with energy. Hence, at  $|t| \approx 1.4 (\text{GeV}/c)^2$  and over a very wide range of energy, we expect  $\frac{d\sigma}{dt} (K^+p)$  at  $|t| \approx 1.4 (\text{GeV}/c)^2$  to be about  $10^{-2} \text{ mb}/(\text{GeV}/c)^2$ . In addition,  $\frac{d\sigma}{dt} (pp)$  is shown in Fig. 3. The latter cross section has been measured<sup>4</sup>

at ISR and is  $\lesssim 10^{-4}$  mb/(GeV/c)<sup>2</sup> at  $|t| \approx 1.4$  (GeV/c). Thus a large difference in the Kp and pp elastic cross sections is expected at  $|t| \approx 1.4$  (GeV/c)<sup>2</sup> at sufficiently high energy. Although this expectation is dependent on our assumed functional form of the eikonal in the pomeron amplitude, we consider this possibility of such a large difference in kaon and proton cross sections to be extremely interesting.

In summary, we predict at NAL energies the following consequences of the Impact Picture:

1. Rather little shrinkage of the K<sup>+</sup>p elastic peak will be observed in distinct contrast to the pp case.
2. The elastic cross section  $\sigma_{el}$  (s) for K<sup>+</sup>p scattering will increase more rapidly than the K<sup>+</sup>p total cross section. Between 50 and 200 GeV,  $\sigma_{el}$  will increase by about 12%.
3.  $d\sigma/dt$  for Kp scattering at  $|t| > 1$  (GeV/c)<sup>2</sup> may be very much larger than the corresponding pp cross section.

Measurements currently in progress at the National Accelerator Laboratory will be able to test these predictions in the very near future.

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Figure 1 The slope parameter,  $b$   $(\text{GeV}/c)^{-2}$  for  $K^{\pm}p$  elastic scattering is plotted versus the square of the centre of mass energy  $s(\text{GeV})^2$ . The curves are for the  $|t|$  intervals indicated. The dashed curves show the results of including only the pomeron term in the amplitude.

Figure 2 The predicted elastic cross section for  $K^{\pm}p$  scattering versus  $s(\text{GeV})^2$ .

Figure 3 The predicted differential elastic cross section for  $Kp$  and  $pp$  scattering at  $s = 400 (\text{GeV})^2$ . Only the pomeron term has been included in the amplitudes.