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Evolution of the Higgs-Yukawa Coupling in the Standard Model

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ABSTRACT

The terms in the two-loop β-function for g_{Higgs} in SU(3) x SU(2) x U(1) gauge theory, involving electroweak corrections, are presented.

Knowledge of the renormalization group behaviour of the couplings in the standard SU(3) x SU(2) x U(1) model is vital when making predictions based on Grand Unified theories about experimentally accessible ("low" energy; < 1 TeV) quantities. In particular, the β-function of g_H , the coefficient in the Lagrangian of the $\phi\bar{\psi}\psi$ term (which gives ψ a "mass" when $\langle\phi\rangle \neq 0$), will influence the predictions for g_b/g_τ and for $\sin^2\theta_W$.[1] This same β-function can be used to find pseudo fixed points, that is, values of g_H(and hence of M_{quark}) at low energies that are relatively independant of g_H at M_X. [2] For the latter purpose, the behaviour of β for $g_{Higgs} > 1$ becomes important, so that it becomes vital to know the full two-loop β-function. Also if very heavy fermions exist, then these two-loop terms can affect mass ratios, $\sin^2\theta_W$, and M_X. [1]

For these calculations, we have assumed that we are at energies much greater than the masses of the particles involved. Thus there will be no mass threshold effects included. These calculations were done in an arbitrary α-gauge, and for both MS and $\overline{MS}$ renormalization schemes. The results are as follows:

For the top quark of a doublet,

$$\beta_T \equiv \frac{dg_T}{d\ell nQ} = \beta_T^{(1)} + \tilde{\beta}_T^{(2)} +$$

$$\frac{g_T}{(16\pi^2)^2}\Big[\frac{225}{16} g_T^2 g_2^2 + \frac{131}{16} g_T^2 g_1^2$$

$$+ \frac{99}{16} g_B^2 g_2^2 + \frac{7}{48} g_B^2 g_1^2$$

$$+ \frac{15}{8} g_E^2 g_2^2 + \frac{25}{8} g_E^2 g_1^2$$

$$+ \frac{45}{8} g_c^2 g_2^2 + \frac{85}{24} g_c^2 g_1^2 + 9\, g_3^2 g_2^2$$

$$+ \frac{45}{8} g_s^2 g_2^2 + \frac{25}{24} g_s^2 g_1^2 + \frac{19}{9} g_3^2 g_1^2$$

$$+ \Big(N_G + \frac{N_H}{2} - \frac{37}{4}\Big) g_2^4 - \frac{3}{4} g_2^2 g_1^2$$

$$+ \Big(\frac{145}{81} N_G + \frac{10}{27} N_H - \frac{53}{216}\Big) g_1^4\Big] \tag{1}$$

where $\beta^{(1)}{}_T$ is the one-loop β-function [3]

$$\beta_T^{(1)} = \frac{g_T}{16\pi^2}\Big[\frac{9}{2} g_T^2 + \frac{3}{2} g_B^2 + g_E^2 + 3g_c^2 + 3g_s^2$$

$$- 8\, g_3^2 - \frac{9}{4} g_2^2 - \frac{17}{12} g_1^2\Big] \tag{2}$$

$\tilde{\beta}^{(2)}$ is the two-loop β function not involving g_2 or g_1 (given in reference [4]); g_3, g_2 and g_1 are the SU(3), SU(2) and U(1) coupling constants; and g_T, g_B, g_E, g_c, and g_s are the Higgs-Yukawa couplings to the top and bottom quarks, the up and down quarks of a different generation, and a heavy lepton, respectively. N_G is the number of fermion generations, and N_H is the number of Higgs doublets (only one of which couples to the fermions to give them masses).

For the bottom quark of a doublet

$$\beta_B \equiv \frac{dg_B}{d\ell nQ} = \beta_B^{(1)} + \bar{\beta}_B^{(2)} +$$

$$\frac{g_B}{(16\pi^2)^2}\Big[\frac{225}{16}g_B^2g_2^2 + \frac{79}{16}g_B^2g_1^2 + \frac{99}{16}g_T^2g_2^2$$

$$+ \frac{91}{48}g_T^2g_1^2 + \frac{15}{8}g_E^2g_2^2 + \frac{25}{8}g_E^2g_1^2$$

$$+ \frac{45}{8}g_s^2g_2^2 + \frac{25}{24}g_s^2g_1^2 + \frac{45}{8}g_c^2g_2^2$$

$$+ \frac{85}{24}g_c^2g_1^2 + 9g_3^2g_2^2 + \frac{31}{9}g_3^2g_1^2$$

$$+ \left(N_G + \frac{N_H}{2} - \frac{37}{4}\right)g_2^4 - \frac{9}{4}g_2^2g_1^2$$

$$+ \left(\frac{-5}{81}N_G + \frac{7}{108}N_H - \frac{101}{216}\right)g_1^4\Big] \tag{3}$$

where

$$\beta_B^{(1)} = \frac{g_B}{16\pi^2}\Big[\frac{9}{2}g_B^2 + \frac{3}{2}g_T^2 + g_E^2 + 3\,g_c^2 + 3\,g_s^2$$

$$- 8\,g_3^2 - \frac{9}{4}g_2^2 - \frac{5}{12}g_1^2\Big] \tag{4}$$

And for a hypothetical heavy lepton,

$$\beta_E \equiv \frac{dg_E}{d\ell nQ} = \beta_E^{(1)} + \bar{\beta}_E^{(2)} +$$

$$\frac{g_E}{(16\pi^2)^2}\Big[\frac{165}{16}\, g_E^2 g_2^2 + \frac{179}{16}\, g_E^2 g_1^2 + \frac{45}{8}\, g_T^2 g_2^2$$

$$+ \frac{85}{24}\, g_T^2 g_1^2 + \frac{45}{8}\, g_B^2 g_2^2 + \frac{25}{24}\, g_B^2 g_1^2$$

$$+ \frac{15}{8}\, g_\mu^2 g_2^2 + \frac{25}{8}\, g_\mu^2 g_1^2$$

$$+ \left(N_G + \frac{N_H}{2} - \frac{37}{4}\right) g_2^4 + \frac{9}{4}\, g_2^2 g_1^2$$

$$+ \left(\frac{55}{9}\, N_G + \frac{13}{12}\, N_H - \frac{3}{8}\right) g_1^4\Big] \tag{5}$$

where

$$\beta_E = \frac{g_E}{16\pi^2}\Big[\frac{5}{2}\, g_E^2 + 3\, g_T^2 + 3\, g_B^2 + g_\mu^2 - \frac{9}{4}\, g_2^2 - \frac{15}{4}\, g_1^2\Big] \tag{6}$$

Here, g_μ refers to a lepton of a dfferent generation.

These results differ from those in reference [5] , where the anomalous dimension of a fermion mass operator is found, in the sectors involving g_1 and g_2 . At high energy, however, one must treat the problem of fermion mass evolution as that of the renormalization of a Yukawa coupling constant (as we do) and thus include the contributions from the anomalous dimension of the scalar particle and from diagrams where the incoming Higgs emits a vector boson that is absorbed by a fermion. That the anomalous dimension of the mass operator is not a physically meaningful quantity can be seen from the fact that the g_2^4 terms are gauge dependant.

Details of these calculations, done in an arbitrary gauge, and the (numerical) physical consequences of the 2-loop corrections appear in reference [6]. For "non-heavy" (< 200 GeV) quarks, the corrections to $\sin^2\theta_W$ and to the proton lifetime will be very small.

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