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PRIMORDIAL INFLATION AND THE MONOPOLE PROBLEM

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ABSTRACT

We discuss the cosmological abundance of magnetic monopoles in locally supersymmetric GUTs and primordial inflation. Depending on the temperature scale of the GUT phase transition (Λ_5 in this model) monopoles may or may not be suppressed sufficiently to satisfy cosmological and astrophysical limits. For example, if the GUT transition occurs after inflation ($\Lambda_5 < T_H \sim 0\ (10^{10} - 10^{11}$ GeV), where T_H is the temperature at which inflation occurs) too many monopoles will be produced unless $\Lambda_5 < 10^9$ GeV. Even then, although the cosmological density limits are satisfied, neutron star limits on the monopole abundance may rule this situation out. If on the other hand $\Lambda_5 > T_H$, SU(5) breaking may occur during inflation and hence the monopole abundance is greatly suppressed as it was in non-primordial inflation. We show that both scenarios are possible with the latter ($\Lambda_5 > T_H$) being preferred for monopole suppression.

Phase transitions in grand unified theories (GUTs) have drastically changed the scenario for the very early Universe by providing mechanisms for baryon generation[1] and inflation[2] with all its benefits. However, many of these phase transitions also produce magnetic monopoles[3], and their history has been the cause of a number of worries to cosmologists. One of the benefits of new inflation[4,5] was to solve the magnetic monopole problem[6]. However, as models of inflation have become more complicated the GUT transition and the inflation transition have become separate phenomena in many models[7]. As a result the magnetic monopole problem has reappeared. In this paper we discuss how the magnetic monopole problem may be solved (or not solved) in variants of broken N=1 supergravity primordial inflation[8,9]. We begin by reviewing the monopole problem and its solution in inflationary models. We then discuss difficulties with a recent suggestion by Linde[10] to solve the problem in primordial inflation. Finally we suggest some modifications to Linde's arguments that may be more successful.

The monopole problem[5] arises whenever a simple group breaks to one containing an explicit U(1) factor[3], such as $SU(5) \rightarrow SU(3) \times SU(2) \times U(1)$. In general, it is difficult to calculate the precise number of monopoles produced by a GUT transition. However, for a first order transition one can estimate that there should be roughly one monopole per bubble of the new phase. It has been shown[6] that if the GUT transition takes place at $T_c \sim 10^{15}$ GeV, the number of monopoles will greatly exceed their cosmologically acceptable abundance.

There have been a number of approaches to calculating the monopole abundance[6,11,12] but here we will outline only the simplest (however naive) approach to the problem. In an adiabatically expanding Friedmann-Robertson-Walker Universe, the expansion timescale can be related to the temperature by

$$t = (90/32\ \pi^3\ g_T)^{1/2}\ M_p/T^2 \tag{1}$$

where $M_p \approx 1.2 \times 10^{19}$ GeV is the planck mass and g_T is the total number of relativistic degrees of freedom at the temperature T. Monopoles are produced during a GUT phase transition due to a lack of correlation among the Higgs fields driving the transition[3]. These scalars could be correlated over distances of at most the horizon scale, 2t. If we then assume that at least one monopole is produced per horizon volume at temperature T_c then the number density of monopoles is

$$n_m \sim (2t)^{-3} \sim 4 \times 10^3\ T_c{}^6/M_p{}^3 \tag{2}$$

when compared to the number of photons present at T_c

$$n_\gamma \sim (g_T/4)T_c{}^3 \tag{3}$$

the monopole to photon ratio is

$$r \equiv n_m/n_\gamma \sim 10^4 \ (T_c/M_p)^3/g_T \tag{4}$$

This is actually an underestimate[6,11,12], because the Higgs correlation length is probably much smaller than the horizon. A more realistic estimate using one monopole per bubble in a first order transition might be[13]

$$r \sim 10^6 \ (T_c/M_p)^3 \quad . \tag{5}$$

Even if a large number of monopoles are produced their number will be reduced by monopole-antimonopole annihilation until[6,11]

$$r \sim (1/g_T^{\ 1/2})\alpha_5^{\ 3} \ m/M_p \simeq 10^{-10} \tag{6}$$

where α_5 is the SU(5) fine structure constant and $m \approx 10^{16}$ GeV is the mass of the monopole. In some non-standard scenarios[14] the ratio, r, could be further reduced.

To see that there is a problem, one must compare the predicted abundance with the allowed cosmological and astrophysical bounds. The surest limit comes from the overall mass density of the Universe[6]. The energy density stored in monopoles of mass m (in GeV) and density n_m is constrained by

$$\rho_m = m\, n_m < \Omega \rho_c \tag{7}$$

where Ω is the ratio of the total mass density to the critical density $\rho_c = 1.88 \times 10^{-29}\, h_o^2$ g cm^{-3}, $h_o = (H_o/100$ km M_{pc}^{-1} s$^{-1})$ and H_o is the present value of the Hubble parameter. The most conservative bounds on the mass density and the hubble parameter require $\Omega h_o^2 < 4$, however consistency with the age of the Universe requires a tighter limit on $\Omega h_o^2 \lesssim 1/4$ (see ref. 15). The monopole to photon ratio is then constrained by

$$r \lesssim 10^{-8}/m \tag{8}$$

comparing this result with (6) one has a monopole problem.

There are other limits on the monopole abundance which are comparable[16] and stronger[17]. However, these limits are more model dependent and we only quote their results. Limits coming from the survival of galactic magnetic fields[16] can be expressed as

$$r \lesssim 10^{-38}\, m\beta \tag{9a}$$

where β is the monopole velocity (typically $\beta \sim 10^{-3}$). on m and β this limit is comparable to the cosmological limit (8). A much stronger limit comes with the inclusion of baryon number violating interactions around a monopole[18].

Limits coming from neutron stars[17] are roughly

$$r \lesssim 10^{-29}\ \beta/\sigma_o \tag{9b}$$

where σ_o is the magnitude of the baryon number violating cross-section normalized to a typical strong cross-section ($\sigma \sim 4 \times 10^{-28}$ cm^2). If taken seriously, both limits put strong constraints on the primordial abundance of monopoles.

A possible solution to the monopole problem[6] lies in the inflationary Universe scenario[5]. The general idea is that bubbles of the broken symmetric phase get expanded by the inflation. After the phase transition the Higgs coherence length is then much longer than the horizon size. For example, the "new" inflationary scenario[4,5] offers the possibility that there is only a single monopole inside the visible portion of the Universe. This is because in these models the whole Universe originates from a single bubble during the GUT phase transition. Unfortunately, these models cannot produce the perturbation spectrum necessary to explain the large scale structure of the Universe[19].

A class of supersymmetric models known as primordial inflation[7,8,9] offers the possibility of explaining the large scale structure of the Universe as well as having the benefits of inflation. We now wish to discuss the monopole problem in the context of primordial inflation. In these models, inflation is no longer associated with the breakdown of SU(5), but rather, it is due to a phase transition involving an SU(5) singlet, the inflaton, ϕ. In such models, it is possible to write down a single superpotential, f, which will describe all scalar interactions and the evolution of the Universe from the planck time till the present. However, because inflation has been separated from SU(5) breaking, one must take special care that the number of monopoles produced during the SU(5) transition still satisfies the cosmological and astrophysical limits. For example, in the models[8,9] utilizing N=1 supergravity, inflation occurs at $T_H \sim 10^{11}$ GeV, where $T_H = H/2\pi$ and H is the Hubble parameter. The model can be arranged so that SU(5) is broken at $T \sim 10^9$ GeV and, hence, after inflation has occured. In this case, the number of monopoles will not be inflated away. Although $r \sim 10^6 \ (T_c/M)^3 \sim 10^{-25}$ might satisfy the cosmological density limit (8), it does not come close to the neutron star limit (9b). Hence there may still be a monopole problem. To illustrate these remarks we would like to work with the following model[8,9,20]. The superpotential is divided into three main parts

$$f = f_I + f_5 + f_B \tag{10}$$

where f_I solely accounts for inflation[8]

$$f_I = m_I^3 \left[\sum_{n=0}^{\infty} (\lambda_n/(n+1)) \, (\phi/M)^{n+1} + \lambda' \right] \tag{11}$$

where ϕ is the inflaton and m_I is an overall scale for the superpotential and $M = M_p/\sqrt{8\pi}$. To get sufficient inflation, couplings λ_i are all $O(10^{-3})$-$O(1)$ with $m_I \sim O(10^{-2})M$. The f_5 part accounts for the breaking of SU(5)[20] and local supersymmetry[21]

$$f_5=(a_1/M)X^4+(a_2/M^2)X^2\mathrm{Tr}(\Sigma^3)+h(z) \qquad (12)$$

where Σ is the adjoint, X and z are SU(5) singlets and the couplings $a_i \sim O(1)$. The function h(z) is normally taken to be

$$h(z)=\mu^2(z+\Delta) \qquad (13)$$

and is the hidden sector used to break local supersymmetry. This simple form for h is known[22] to have problems. After inflation the z field is not exactly at its global minimum and it is difficult to dissipate the energy stored in the z field. More complicated forms for h may[23] resolve this problem. Finally, f_B is used to account for baryon generation[9,20]. The details of f_B are not relevant for this discussion.

To analyze the behavior of the model, one must write down the scalar potential in terms of f. For N=1 supergravity[24], the minimal expression is

$$V(y_i)=e^{\Sigma|y_i|^2/M^2}[|f_{y_i}|^2 - 3|f|^2/M^2] \tag{14}$$

where

$$f_{y_i} = \partial f/\partial y_i + y^* f/M^2 \tag{15}$$

for each chiral supermultiplet y_i. In this model (Eqns. 10-12) one can separate out, to a large extent, the inflaton self-interactions from the rest of V. One then has a scalar potential $V(\phi,\phi^*)$ in terms of the λ_i which will be required to meet the conditions of sufficient inflation[7)]. The goal is to have a potential with a barrier near the origin while still being very flat to allow a long roll over time scale to a local minimum at $\langle\phi\rangle \sim M$. Recently it has been pointed out[25,5)] that the effects of finite temperature[8)] render this scheme inconsistent by proving the existence of a local minimum at some $\phi_o < M$ with $V(\phi_o)<0$. Thus the inflaton would produce a negative[25)] cosmological constant. However, the finite temperature corrections used were computed neglecting the effects of other chiral supermultiplets in the model. It turns out that these are dominant and a suitable change of signs of the couplings λ_i (among other possible variations) will negate the existence of the troublesome minimum. It should also be noted that trying to use this minimum as a broken supersymmetric minimum with zero cosmological constant as in Ref. 25 would be disastrous. Supersymmetry would then be broken at a scale $M_S \sim 10^{16}$ GeV producing a gravitino of mass $m_{3/2} \sim 10^{13}$ GeV and thus destroying the gauge hierarchy and one of the original motivations for supersymmetry.

Using the procedure of eqs. 14 and 15, we can write down the scalar potential for X and Σ. Before looking at this, it is important to realize that the breaking of SU(5) in supersymmetric theories is complicated by the existence of several degenerate vacua[26]. At zero temperature in globally supersymmetric models there are at least three degenerate minima, SU(5), SU(3) x SU(2) x U(1) and SU(4) x U(1). At finite temperature, the degeneracy is broken by the differing number of particle degrees of freedom in each phase, with SU(5) being the lowest minimum. This presents a problem as to how one ever gets out of SU(5). To accomplish this it has been observed[26,27] that when the temperature drops below Λ_5 (defined to be the scale at which the GUT fine structure constant, α_5, becomes O(1)) the above picture breaks down. At $T \sim \Lambda_5$ strong coupling phenomena decreases the effective number of degrees of freedom in the SU(5) phase. This effect may push the SU(5) minimum above the others. The tunnelling rate out of SU(5) will be vanishingly small unless the barrier separating these phases is kept small[28]. This can be done by either introducing a small coupling[27] $\lambda \sim 10^{-14}$ or by looking at N=1 supergravity[26] with couplings of order unity.

In the superpotential (10), f_5 (12) will accomplish the above without small couplings. The scalar potential for (14) can be simplified to

$$V/M^4 = |\chi^3+\chi\sigma^3|^2 + |\chi^2\sigma^2|^2 + \varepsilon\,(\chi^4+\chi\sigma^3+h.c.) + \varepsilon^2(|\chi|^2+|\sigma|^2) \qquad (16)$$

where $\chi = X/M$; $\sigma = \Sigma/M$ and $\varepsilon = (\mu/M)^2 \approx 10^{-16}$ for a supersymmetry breaking scale $\mu \sim 10^{10}$ GeV. In the analysis of this potential (16), it was found [20] that the global minimum is an SU(3) x SU(2) x U(1) symmetric minimum with

$$\langle\Sigma\rangle = \varepsilon^{1/4}M = (\mu M)^{1/2} \sim 10^{15}\ \text{GeV} \qquad (17a)$$

and

$$\langle X\rangle = \varepsilon^{3/8}M = 10^{-6}M \qquad (17b)$$

and hence also predicts the GUT scale from the planck scale and the supersymmetry breaking scale μ. (The weak scale is governed by the gravitino mass scale which is $\varepsilon M \sim \mu^2/M \sim 10^2$ GeV). Furthermore, the barrier height is only $\varepsilon^{5/2}M^4 \sim (10^8\ \text{GeV})^4$, without small couplings. It is possible, therefore, that if $\Lambda_5 > 10^8$ GeV, strong coupling phenomena will be able to drive the transition.

In the past, we have discussed values of $\Lambda_5 \sim 0(10^9$ GeV). The goal was to push Λ_5 (and hence T_c) as low as possible to produce few monopoles. For example, with this value of Λ_5, we might expect a monopole abundance $r \sim 0(10^{-26})$. This is sufficient for the cosmological (8) and galactic magnetic field (9a) limits, but falls six orders of magnitude short of the neutron star limits (9b). In fact, to make r consistent with (9b) one would need $T_c \sim \Lambda_5 \sim 10^7$ GeV. However, this value could no longer drive a phase transition over a barrier of 10^8 GeV and would also begin to make baryon generation very difficult.

Before proceeding, we note that in this scenario the breaking of SU(5) occurs after the exponentially expanding phase. Although $\langle\phi\rangle \approx M$, the hubble parameter at the time of inflation is

$$H^2 \approx 1/3\ m_I^6\ \lambda_o^2 \sim 5 \times 10^{-14}\ M^2 \tag{18}$$

$$H \sim 5 \times 10^{11}\ \text{GeV}$$

Thus inflation takes place at $T_H \sim 10^{11}$ GeV, i.e., before the breaking of SU(5) at $T_c \sim \Lambda_5$.

Recently Linde[10)] has argued that SU(5) will break during primordial inflation (even if $T_H > \Lambda_5$) and as a result the monopole abundance will dilute away exponentially. We believe that although the general idea is a good one the mechanism is not correct. Linde's argument depends on scalar field fluctuations in deSitter space[29]. Define the total mass^2

$$D = M_o^2 + cT^2 + 12\lambda^2\langle\Sigma^2\rangle \qquad (19)$$

where M_o^2 is a bare mass and cT^2 is the Σ^2 coefficient of the temperature correction to the effective potential

$$V_T = T^2/8\ \partial^2V/\partial\Sigma\partial\Sigma^* \qquad (20)$$

and λ is the Σ^4 coupling. In the limit $D \ll H^2$, one finds that the fluctuations in the adjoint scalar field are [29]

$$\langle\Sigma^2\rangle \sim H^4/D \qquad (21)$$

which must then be solved self consistently. Linde argues that D is small, so that $\langle\Sigma^2\rangle$ is large enough to populate the different symmetry breaking minima. The probability of finding our universe to be in the 3-2-1 phase is then a fraction of O(1).

We believe that D is not small. In (19) the $\lambda^2\langle\Sigma^2\rangle$ term is negligible compared to the other terms because λ is so small in the effective potential derived from (12). (The Σ^4 term is in fact negligible to the terms included in (16) and for that reason left out.) The M_o^2 term is also very small with a value given by

$$M_o{}^2 \sim \mu^4/M^2 \sim (100 \text{ GeV})^2 \tag{22}$$

The temperature correction term comes from two sources. The first term arises from the F term in the potential. It is small because it is proportional to λ. However, there is a second contribution coming from the D-term and it is proportional to α_5. Therefore, the fluctuations are of order

$$\langle\Sigma^2\rangle \sim H^4/\alpha_5 \; T_H{}^2 \sim 4\pi^2 H^2/\alpha_5 \tag{23}$$

These are not large enough to drive the transition.

The solution to the monopole problem can still be realized if we can raise the value of Λ_5 and hence T_c to a value $\gtrsim T_H$. In fact, by modifying the Higgs sector and by choosing slightly different initial parameters one can find a value $\Lambda_5 \sim 10^{12}$ GeV. With this value of Λ_5, the SU(5) transition will take place during the epoch of primordial inflation [7-10] and the monopoles are inflated away. The transition may take place in two ways; first, as previously described[20], via strongly coupling effects. In that case strong coupling effects are of order $\Lambda_5{}^4$ near the origin. We[30] warn the reader that the potential (16) is now altered by the fact that ϕ is not at its global minimum. The effect is to raise the barrier from $\epsilon^{5/2}M^4$ to $(f_I(0)/M^3)^{5/2}\ M^4$. In this model $f_I(0) \sim m_I{}^3\lambda$ and must hence be constrained so that

$$(f_I(0)/M^3)^{5/8} < \Lambda_5/M. \qquad (24)$$

(Note that the barrier is not affected by the D-term finite temperature correction, as the barrier is at $\Sigma >> T_H$ or Λ_5. The correction is cut off by $e^{-\Sigma/T}$.) Another possibility is present for the SU(5) transition in the case of primordial inflation. Suppose strong coupling phenomena do not drive the transition, but instead Σ finds itself in a new minimum near the origin. Depending on the parameters, if the tunnelling rate through this barrier (the tunnelling rate is still very low) is greater than the rate for forming a bubble due to the inflation transition, one could simply wait inside an SU(3) x SU(2) x U(1) bubble for inflation to occur. So long as the probability of tunnelling to the 3-2-1 phase is comparable to the 4-1 phase one does not have to worry about which one is closest[31)].

To summarize, we have seen that contrary to past fears, primordial inflation coupled to supersymmetric SU(5) breaking can cure the monopole problem. Independent of whether the SU(5) transition proceeds via strong coupling phenomena or tunnelling the monopole abundance is exponentially reduced if $\Lambda_5 > T_H$. For 10^8 GeV $< \Lambda_5 < 10^9$ GeV the monopole abundance is low enough to pass the cosmological density limits but probably not the neutron star limits. Other versions[3] of supersymmetric SU(5) breaking in which the SU(5) transition takes place before primordial inflation offer the same types of solution.

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