

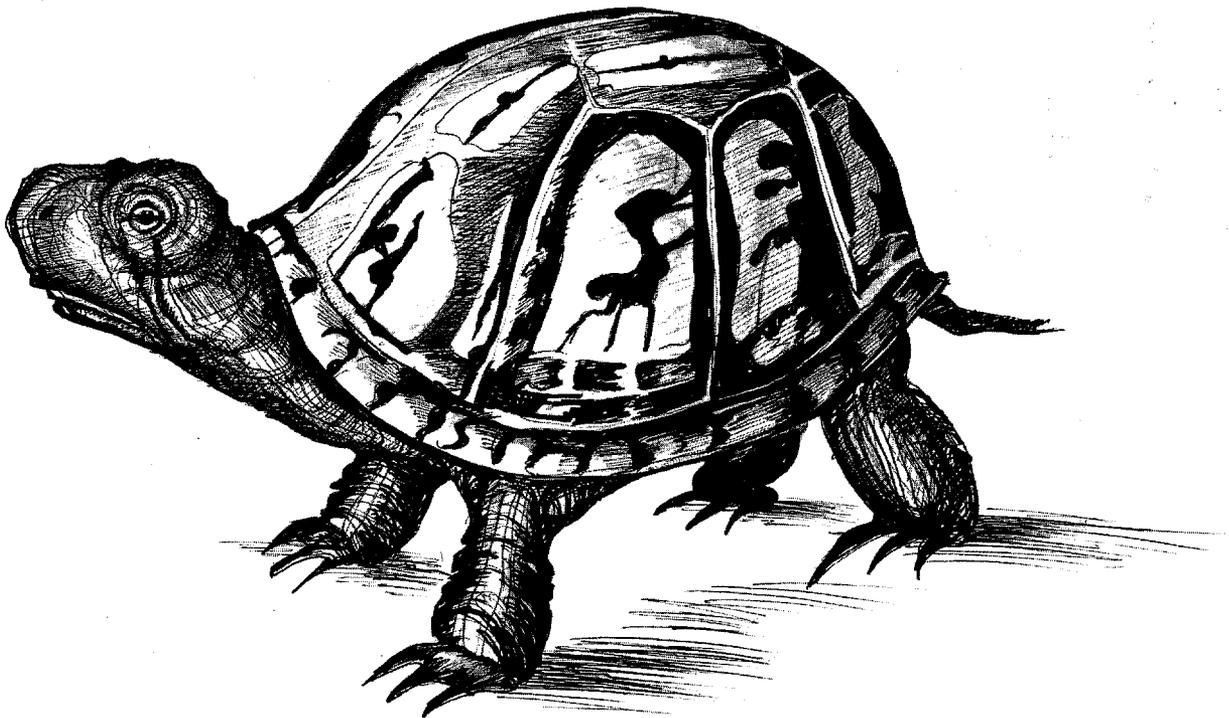


**TURTLE**  
(Trace Unlimited Rays Through Lumped Elements)

A Computer Program For Simulating Charged  
Particle Beam Transport Systems

David C. Carey

December, 1971



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ABSTRACT

TURTLE is a computer program designed to simulate charged particle beam transport systems. It allows evaluation of the effect of aberrations which exist in beams with small phase space volume. These include higher order chromatic aberrations, effects of non-linearities in magnetic fields, and higher order geometric aberrations due to the accumulation of second order effects. The beam at any point in the system may be represented by one and two-dimensional histograms.

## I. INTRODUCTION

TURTLE is a computer program useful for determining many characteristics of a particle beam once an initial design has been achieved. Charged particle beams are usually designed by adjusting various beam line parameters to obtain desired values of certain elements of a transfer or beam matrix. Such beam line parameters may describe certain magnetic fields and their gradients, lengths and shapes of magnets, spacings between magnetic elements, or the initial beam accepted into the system. For such a purpose one typically employs a matrix multiplication and fitting program such as TRANSPORT.<sup>1</sup> TURTLE is designed to be used after TRANSPORT. For convenience of the user their input formats have been made compatible.

The use of TURTLE should be restricted to beams with small phase space. The lumped element approximation, described below, precludes the inclusion of the effect of conventional local geometric aberrations (due to large phase space) of third and higher order. A reading of the discussion below will indicate more clearly the exact uses and limitations of the approach taken in TURTLE.

## II. GENERAL THEORY

### A. Uses and Limitations of the Matrix Approach

We can represent the position and direction of travel of a particle entering a beam line via a vector with six coordinates<sup>2,3</sup>

$$x = \begin{pmatrix} x \\ \theta \\ y \\ \phi \\ \ell \\ \delta \end{pmatrix} \quad (1)$$

The coordinates  $x$  and  $y$  represent respectively the horizontal and vertical displacements at the position of the particle,  $\theta$  and  $\phi$ , the angles with the axis of the beam line in the same planes. The quantity  $\ell$  represents the longitudinal position of the particle relative to a particle traveling on the magnetic axis of the system with the central momentum designed for the system. The remaining quantity  $\delta = \frac{\Delta p}{p}$  gives the fractional deviation of the momentum of the particle from the central design momentum of the system.

The effect of the passage of a particle across a magnetic element or a drift space may be represented to first order by a transfer matrix  $R$ . The coordinates  $x(1)$  of the particle at the end of the element are then given in terms of those at the beginning  $x(0)$  as

$$x(1) = Rx(0). \quad (2)$$

The effect of successive elements, each with its own R matrix may be given by a total R matrix  $R(t)$  which is equal to the product of the individual R matrices.

$$R(t) = R_n R_{n-1} \dots R_2 R_1 \quad (3)$$

The first columns of such a matrix are obtained by solving a differential equation of the form

$$g'' + k^2 g = 0 \quad (4)$$

with appropriate boundary conditions, and taking either the value or the derivative of the solution. The differentiation is with respect to distance along the beam line and  $k^2$  is a function only of that quantity. The last two columns are obtained by solving the equation

$$g'' + k^2 g = f \quad (5)$$

where  $f$  is a driving term given in terms of quantities which depend only on the position along the beam line and single factors of solutions to either equation (4) or (5). Equation (5) may be solved by a Green's function by writing

$$g(t) = \int_0^t G(t, \tau) f(\tau) d\tau. \quad (6)$$

If we treat equation (2) as the first term of a power series expansion of  $x(l)$  in terms of  $x(0)$  we may write further terms as follows:

$$x_i(t) = \sum_j R_{ij} x_j(0) + \sum_{j,k} T_{ijk} x_j(0) x_k(0). \quad (7)$$

Here our definition of the T matrix differs from Brown's<sup>2</sup> in that ours is rectangular while Brown's is triangular. The use of a rectangular matrix is more easily extended to the study of higher order effects. The difference between the two matrices is that the off diagonal elements of the rectangular matrix are exactly half those of the triangular matrix.

As in the case of the R matrix, the T matrix, for a succession of elements may be written in terms of the R and T matrices of the individual elements via a product. The T matrix  $T(t)$  for a succession of two elements is written in terms of the individual R and T matrices as follows:

$$T(t)_{ijk} = \sum_{\ell} R_{i\ell}^{(2)} T_{\ell ik}^{(1)} + \sum_{\ell m} T_{ilm}^{(2)} R_{\ell j}^{(1)} R_{mk}^{(1)} \quad (8)$$

Notice that at no point in the evaluation of a T matrix for a beam line are two individual T matrices multiplied together. Such a multiplication would yield terms of order higher than second. Since in a matrix approach one characterizes the effects of the beam line according to order, and we are here expanding to second order, we must consistently truncate our results to second order.

In a matrix approach the beam itself may also be represented by a matrix  $\sigma$ . The phase space occupied by the beam is taken to be an ellipsoid in six dimensions. Usually this matrix is initially taken to be diagonal

with the square roots of the diagonal elements being equal to the semi-major axes of the ellipsoid. The beam ellipsoid at a later point is obtained, using the R matrix, as

$$\sigma_1 = R \sigma_0 R^T. \quad (9)$$

The diagonal elements are now the squares of half the maximum extent of the ellipsoid in a given dimension. The off diagonal elements give the correlation between the coordinates, as for a tilted ellipse. A phase space envelope which is initially ellipsoidal continues to be so only when we limit our consideration to first order effects. When higher orders are considered the elements of the beam matrix may be taken as second moments of a distribution, but give us no information as to the actual shape of the phase space envelope.

The matrix method is indispensable for fitting and is the only reasonable way to obtain an initial design of a beam. It is also unsurpassed as a tool to determine individual second order influences on particle trajectories and minimizing such aberrations. Its limitations are in the representation of the beam phase space and the fact that the results are limited by the order of the Taylor's series expansion.

### B. Ray Tracing Methods

A true ray tracing program computes the trajectory of a particle through a magnetic field directly. It, therefore, does not distinguish among different order effects on a ray, but represents all orders to the accuracy of the numerical integration of the equations of motion. Such an approach is also useful for determining individual contributions to aberrations. If, however, one wishes to represent the phase space occupied by the beam, it is necessary to run large numbers of rays through the system to obtain a reasonable population. For a complicated system this approach can prove time consuming. In addition it is often unnecessary, as in the case of beams possessing a small phase space volume.

### C. Theory of Ray Tracing Through Lumped Elements

In order to describe the procedure employed in TURTLE, we must explain what is meant by the distinction between local and global classification of aberrations according to order. If we once again let  $\bar{x}$  be a vector giving the coordinates and direction of a particle (henceforth referred to as a ray), and expand the differential equation of motion of a particle passing through a magnetic field in powers of  $x$ , we obtain, deleting subscripts,

$$Dx + Ex^2 + Fx^3 + \dots = 0. \tag{10}$$

The coefficients D, E, and F are matrices, with the diagonal terms of D being differential operators

$$Dx = \left(\frac{d^2}{dt^2} + k^2\right)x. \tag{11}$$

The first order transfer matrix R is obtained by solving the differential equation to first order, i.e.,

$$Dx = 0 \tag{12}$$

The second order transfer matrix T is now obtained from the second order coefficients in the differential equation via a Green's function

$$T(t,0) = \int_0^t G(t,\tau) E(\tau) x^2(\tau) d\tau \tag{13}$$

where  $x(t)$  is obtained from the first order transformation

$$x(t) = R(t,0)x(0). \tag{14}$$

We see that to second order, the coefficients of a given order in the differential equation of motion give rise to transfer matrices of the same order. If, however, we wish to go further and ask for third order transfer matrices  $U_{ijkl}$ , then we find that

$$\begin{aligned}
 U = & \int_0^t G(t,\tau) F(\tau) x^3(\tau) d\tau \\
 & + \int_0^t G(t,\tau_1) E(\tau_1) x(\tau_1) \int_0^{\tau_1} G(\tau_1,\tau_2) E(\tau_2) x^2(\tau_2) d\tau_2 d\tau_1 \\
 & + \int_0^t G(t,\tau_1) E(\tau_1) \int_0^{\tau_1} G(\tau_1,\tau_2) E(\tau_2) x^2(\tau_2) d\tau_2 x(\tau_1) d\tau_1.
 \end{aligned}$$

The first term contains the third order coefficients in the differential equation while the second and third contain the second order coefficients. Elements of transfer matrices beyond second order therefore involve not only coefficients of the differential equation of the same order, but also accumulations of lower order terms. We accordingly, call a classification by order of aberrations according to their appearance in the equation of motion as a local classification. A classification of terms by appearance in a transfer matrix will be called global.

A matrix approach as defined in part (A) classifies aberrations by order both locally and globally. Such an approach is necessary for any order if one is interested in correcting aberrations of that order. A true ray tracing program as explained in B need not classify aberrations by order of all.

In the lumped element approach to ray tracing, we classify aberrations by order locally but not globally. The passage of a ray across an individual element is given by a transformation which yields the output ray directly from the input ray. A large number of rays can then be passed through a system in a short time. The coordinates of the rays can then be collected at any point in the beam line and histograms can be generated.

We can then exhibit the phase space occupied by the beam explicitly, and do not have to depend on an ellipsoid formalism.

The use of the lumped element approach permits the inclusion of many sorts of aberrations, but effectively precludes the use of others. To understand how TURTLE works we must examine the different types of local aberrations. We characterize local aberrations into four types: chromatic effects, geometric effects, magnetic field effects, and mixtures of any of the first three types.

Chromatic effects are due to a deviation of the momentum of a ray from the central momentum of the beam line. An example would be the chromatic aberration of a quadrupole, where the focussing strength depends on the momentum of the particle. Geometric effects are associated with the phase space accepted by the beam line. A trajectory entering a bending magnet at an angle to the central trajectory traverses a different path length in the field and is therefore bent through a different angle than is the central trajectory. This effect gives rise to second and higher order geometric aberrations for bending magnets. Another example occurs in the normal treatment of quadrupoles. The harmonic oscillator approximation for the motion of a charged particle in a quadrupole is based on a small angle approximation. Corrections for this approximation involve geometric aberrations of third

and higher order. Magnetic field effects are due to non-linearities appearing in the expansion of the magnetic field in terms of the transverse coordinates of the beam line. An example is the effect of a sextupole for on-momentum rays. Mixed effects represent combinations of the above effects. One example might be the momentum dependence of the effect of a sextupole.

The intent of creating TURTLE was to examine chromatic aberrations and the effect of non-linearities in magnetic fields to all orders, and to evaluate the effect of slits and apertures and to represent the beam phase space distribution, including effects of second and higher orders. Geometric effects are considered locally only to second order, but higher order global effects will appear due to the accumulation of second order effects. It was not possible to achieve this goal rigorously so we explain below what was done in each case, and why the approximations used should be valid for beams possessing a small monoenergetic phase space volume.

Transfer matrix elements for quadrupoles and sextupoles are evaluated directly for each ray from the actual momentum of the ray, and are, therefore, exact to all orders in chromatic effects. However, in bending magnets, chromatic effects are evaluated only to second order.

But in high energy separated function beams the net focussing effects of bending magnets are small compared to quadrupole contributions, so it is usually sufficient to include only second order chromatic corrections for bending magnets.

Because of the small phase space volume occupied by typical high energy beams, the third and higher order geometric effects of a given element will be small. However, higher order global terms may occur due to cumulative effects of second order terms. The large lever arms for aberrations in beams at high energies will enhance the importance of such cumulative effects relative to local higher order geometric effects.

The inclusion of apertures and slits and the representation of the beam phase space with histograms allow a further step in the realistic representation of a beam.

Below we explain how to use the program and further describe each element available.

### III. USE OF TURTLE

TURTLE is designed to be run using the same deck of data cards as was used for TRANSPORT. Typically one will start with a deck of cards containing approximate values of the final parameters to use in obtaining a fit to the desired constraints. Once the final set of parameters is known, one alters this deck to include them for purposes of studying second order aberrations and misalignments. With a few changes this deck may be used as input for TURTLE. The data format is free field and the deck structure is the same. Below we discuss the deck structure and each of the type codes. For completeness we include many things that are unchanged from TRANSPORT.

A. Structure of the Deck

1. Title Card

The first card of the deck contains the title of the run enclosed in single quotes. No other item should appear on this card.

2. Number of Rays

The second card in a TRANSPORT deck contains an integer which serves as an indicator. When using TURTLE this card contains an integer indicating how many rays one wishes to run through the system. Naturally one will want to choose this number sufficiently high to obtain good statistics. A number of rays equal to several thousand should be quite reasonable for any beam. The only limitation on the number of rays that can be run will be the use of computer time. The user will eventually wish to select this number on the basis of his own experience.

3. Data

The elements with their type codes and appropriate parameters and labels are entered in sequence just as in TRANSPORT. Each element must be followed by a semicolon. The labels are not used by TURTLE but may be retained and will appear in the output. This will enable the user to compare his output from TURTLE with that from TRANSPORT. There are additional type codes indicating the creation of histograms. All type codes, including those for histograms, will be explained below.

4. Comments

Comments may be placed in the data deck before type code entry. They are indicated by enclosing in parentheses.

5. Sentinel

The input data is terminated with a SENTINEL card. The program, after reading and initially processing the data, runs the specified number of rays through the beam line and collects and plots histograms.

B. Type Codes

BEAM - Type Code 1.

Type code 1. can be used either to represent the initial phase space of the beam or an RMS addition to the beam. There are either eight or nine parameters on the beam card, as follows:

1. Type code 1.0
2. One half the horizontal extent  $x$  of the beam (cm in standard units)
3. One half the horizontal angular divergence  $x'$  (mr in standard units)
4. One half the vertical extent  $y$  of the beam (cm in standard units)
5. One half the vertical angular divergence  $y'$  (mr in standard units)
6. One half the longitudinal extent of the beam. This parameter is not used by TURTLE but is included in the input for consistency with TRANSPORT
7. One half the momentum spread ( $\delta$ ) of the beam (in units of percent  $\Delta p/p$ )
8. The central design momentum of the beam (GeV/c in standard units)
9. The code digit 0.0 indicating an RMS addition to the beam. If one is specifying the initial phase space this entry must be absent.

Each ray is chosen at random with its coordinates constrained to lie within the limits specified on the beam card. In addition, the x and y coordinates are constrained to lie within an upright ellipse whose semi-axes are the quantities given on the beam card. The angular coordinates x' and y' are also constrained to lie within a similar ellipse. This prevents the implicit representation of a preferred direction in space by the choice of coordinates. In other words, targets may be circular and scattering is isotropic. Further restrictions on the phase space, such as the use of a multi-dimensional ellipsoid were felt not to be physically realistic. This is illustrated by the fact that the target is the same size for particles of different momentum.

If we let  $x_0$ ,  $x_0'$ ,  $y_0$ ,  $y_0'$ ,  $\ell_0$  and  $\delta_0$  represent the maximum extents of the beam envelop in each coordinate, then the additional restriction imposed by TURTLE is equivalent to the two equations

$$\left(\frac{x}{x_0}\right)^2 + \left(\frac{y}{y_0}\right)^2 \leq 1,$$
$$\left(\frac{x'}{x_0'}\right)^2 + \left(\frac{y'}{y_0'}\right)^2 \leq 1.$$

The hypervolume of this four dimensional region is equal to  $\pi^2 x_0 x_0' y_0 y_0'$ . By contrast the hypervolume of the region specified by the single ellipsoidal condition

$$\left(\frac{x}{x_0}\right)^2 + \left(\frac{x'}{x_0'}\right)^2 + \left(\frac{y}{y_0}\right)^2 + \left(\frac{y'}{y_0'}\right)^2 \leq 1$$

is equal to  $(\pi^2/2) x_0 x_0' y_0 y_0'$ , or exactly one half that of the region specified by the first set of conditions. This additional volume occurs near the boundaries of the region specified in two ellipse condition. When the multi-dimensional distribution is projected onto one or two dimensions the ellipsoidal condition yields a distribution which is very sparse near the edges. The two ellipse condition yields a projection where the edges of the distribution are much more sharply defined and is thus much better suited to beam line studies.

For an RMS addition to the beam, the coordinates are chosen in the same manner, but are then added to the coordinates of the ray at the point where the RMS addition is made. Because of the random selection of both the original ray and the addition to it the effective widths of the distributions in any coordinate will add in an RMS fashion. It should be remembered that the parameters on the card indicating the RMS addition specify the maximum amplitude of the scattering. If the element immediately proceeding an RMS addition is a slit (type code 6.0) the RMS addition will be made only if the ray hits the slit. If the ray passes through the slit its coordinates will be unchanged and it will continue unperturbed.

FRINGING FIELDS AND POLE FACE ROTATIONS  
FOR BENDING MAGNETS - Type Code 2.

Type code 2.0 specifies the pole face rotation of a bending magnet and causes the effect of the fringing fields to be calculated. It should immediately precede or follow a type code 4.0 representing a bending magnet. There are two parameters.

1. Type code 2.0
2. The pole face rotation angle (in degrees)

Even if the pole face rotation angle is zero, it is necessary to insert a 2.0 card to take the fringe field into account. When the program is run the effects of the fringe field and the bending magnet are included in a single transfer matrix for the entire magnet. It is therefore not possible to insert a histogram between a 2.0 and a 4.0 element. More information about the transfer matrix will be found in the description of the 4.0 type code.

DRIFT SPACE---Type Code 3.0

A drift space is a region containing no magnetic elements. Two parameters are required.

1. Type code 3.0
2. Length (normal unit is meters)

SECTOR BENDING MAGNET -- Type Code 4.0

For a sector bending magnet the entrance and exit faces of the magnet are straight and perpendicular to the central axis of the beam. Other types of magnets may be represented by using a type code 2.0 and any of the bending magnet parameters included under type code 16.0. A type code 4.0 requires four parameters.

1. Type code 4.0
2. The effective length of the central trajectory through the magnetic field (normal unit is meters)
3. The field strength  $B_0$  along the central trajectory (normal unit is kilogauss)
4. The field gradient  $n$  in dimensionless units.

$$n = \frac{-\rho}{B_0} \frac{\partial B}{\partial x} (x, 0, t)$$

where  $\rho = \frac{P_0}{qB_0}$ , with  $P_0$  the central momentum at the beam and  $q$  the charge of the particle,  $\rho$  is the radius of curvature of the trajectory.

If second order (see type code 17.0) is not specified, a ray with the beam central momentum is transformed through the magnet using the first order transfer matrix  $R$  so that  $x(1) = Rx(0)$ . If the ray does not have the central momentum we use an off momentum  $R$  matrix which is obtained from the ordinary first order matrix  $R^0$  and the chromatic elements of the second order matrix  $T$  via the equation

$$R_{ij} = R_{ij}^O + T_{ij6} \frac{\Delta P}{P}$$

where  $\frac{\Delta P}{P}$  is the percentage deviation from the central momentum. If a type code 17.0 card is included in the deck, second order geometric effects will be included. These correspond to the matrix elements  $T_{ijk}$ , where  $j$  and  $k$  are both equal to or less than four.

To include the effect of fringing fields, it is necessary to include 2.0 type cards before and/or after the 4.0 card. When the program is run it assembles a single transfer matrix for each bending magnet including the fringe fields. It then transforms each ray using this assembled transformation matrix.

QUADRUPOLE - Type Code 5.0

A quadrupole requires four parameters for its specification.

1. Type code 5.0
2. The effective length of the quadrupole field.  
(normal unit is meters)
3. The pole tip field,  $B_0$ , a positive field indicating a horizontally focusing quad.  
(normal unit is kilogauss)
4. Pole tip half aperture  $a$ . The gradient is obtained by dividing the pole tip field by this aperture.

For a perfect quadrupole the first four components of the ray vector  $\bar{x} = (x, x', y, y', \ell, \delta)$  are carried through the quad via the transformation matrix.

$$\begin{pmatrix} \cos k\ell & \frac{1}{k} \sin k\ell & 0 & 0 \\ -k \sin k\ell & \cos k\ell & 0 & 0 \\ 0 & 0 & \cosh k\ell & \frac{1}{k} \sinh k\ell \\ 0 & 0 & k \sinh k\ell & \cosh k\ell \end{pmatrix}$$

where  $k^2 = \frac{qB_0}{ap}$ ,  $p$  is the actual momentum of the ray, and  $q$  is the charge of the particle. This matrix is for a horizontally focussing quadrupole. A matrix for a vertically focussing quadrupole is obtained by exchanging the two submatrices occurring on the diagonal. Since this matrix is evaluated from each ray using the actual momentum of the ray it is good to all orders in chromatic effects.

Higher order multipole effects for a realistic quadrupole may also be included via a 16.0 type code entry. Multipoles up to and including a 40-pole may be included. The multipole aberrations are taken to be lumped at the longitudinal midpoint of the quadrupole. The ray is transformed halfway through the quad, perturbed by the multipole aberrations, then transformed through the remainder of the quad. If no multipole aberrations are included the ray passes through the quad in a single step. The multipole component strengths are normalized to unit gradient and need be entered only once for similar but differently excited quads. The effect of the multipoles is also calculated from the actual momentum of the ray, making this effect correct to all orders chromatically.

SLIT -- Type Code 6.0

A slit requires three parameters

1. Type Code 6.0
2. The coordinate in which the slit appears.  
(e.g. 1. for a horizontal slit)
3. The half opening of the slit. (in the units  
for the appropriate coordinate)

If a ray comes to a horizontal slit and the absolute value of its x coordinate exceeds the half aperture of the slit, the ray will be stopped and the program will proceed to the next ray. If an RMS addition to the beam is specified immediately after the slit, the ray will not be stopped but its coordinates will be changed by an amount within the range specified on the RMS addition. See type code 1.0 for further details.

If the absolute value of the x coordinate of the ray is within the half aperture of the slit, the ray will pass undeflected in all cases. This description applies similarly for a slit in any other coordinate.

SHIFT IN THE BEAM -- Type Code 7.0

The coordinates of a ray may all be shifted by a constant (independent of the ray) amount at a given point in the beam. Seven parameters are needed.

1. Type code 7.0
- 2-7. The amount of the shift in each coordinate. Units are the units used for that coordinate. Coordinate No. 5 is not used but included for compatability with TRANSPORT.

There are no phase space requirements associated with the use of this element. A known misalignment may be simulated by sandwiching an element between two 7.0 cards.

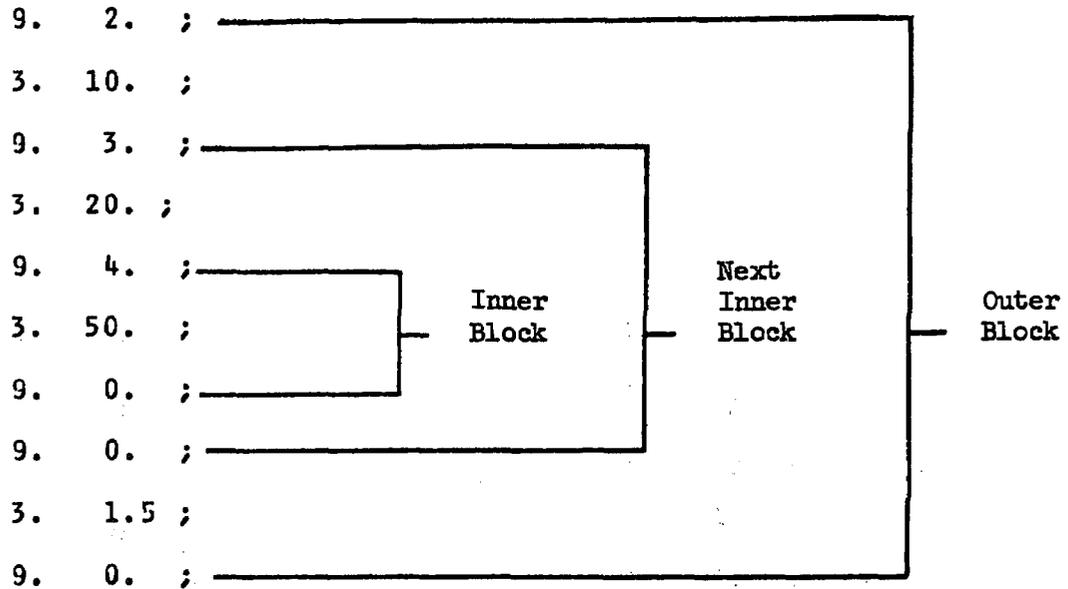
REPEAT -- Type Code 9.0

A section of the beam may be repeated as many times as desired by sandwiching that section between two repeat cards. The two parameters needed are:

1. Type code 9.0
2. No.-of-times section is repeated. Equal to zero for end of a repeated section.

Repeats may be nested four deep. Care should be taken to insure that for each card beginning a repeated section there is one terminating that section.

Example of Nesting:



The total length of this sequence is:

$$2*(10. + 3*(20 + 4* 50) + 1.5) = 1343.$$

APERTURE CONSTRAINTS - Type Code 13.

The insertion of a 13. 10. card causes all specified apertures to be taken into account. If a ray passes outside a given aperture it is terminated and a new ray is initiated at the beginning of the system. The quadrupole half aperture is normally taken to be circular and equal to the fourth entry on the type code 5.0 card. One can specify a different aperture for quads, and make it elliptical through the use of a 16. 100. element. The apertures for bending magnets are given by the 16. 4. and 16. 5. elements. All aperture constraints are applied both at the beginning and the end of the element to which they apply. See the section describing the use of the 16, element for further details on aperture specification.

ARBITRARY TRANSFORMATION MATRIX -- Type Code 14.

An arbitrary first order 6 x 6 matrix transformation may be introduced. There are eight parameters.

1. Type code 14.
- 2-7. The elements of the given row of the transfer matrix.  
The units of these numbers must be chosen to be consistent with the units for the coordinates.
8. The number of the row.

A matrix must be read in one row at a time. An uninterrupted sequence of 14. type cards is taken as providing entries to the same matrix. Any unspecified row is taken to be that of the identity matrix. If two successive sets of 14. cards are to provide elements of successive matrices, they must be separated by a different element. An example of an element having no other effect should be a (3. 0. ;), a drift space of zero length.

Second order terms may be introduced by including the 22 additional numbers.

9. Continuation code 0.
- 10-30. The 21 second order matrix elements  
T(i11) T(i12) T(i13) T(i14) T(i15) T(i16) T(i22)  
T(i23) T(i24) T(i25) T(i26) T(i33) T(i34) T(i35)  
T(i36) T(i44) T(i45) T(i55) T(i56) T(i66)

in the order given. The letter "i" indicates the row number, and specifies the coordinate to which these matrix elements contribute.

Each set of second-order coefficients accompanies the row of the first order matrix which contributes to the same coordinate. Once again successive 14. elements are taken as giving entries into the same R and T matrices. Any unspecified row is taken as being the same as the identity transformation. This means that the R matrix elements are given by the kronecker delta and the T matrix elements are all zero.

UNITS CHANGES -- Type Code 15.

Units may be changed to any desired. A specification of units change should appear before any other type code in the deck. All subsequent input data should then be consistent with the units used. There are four entries on a 15. card.

1. Type code 15.
2. A code digit indicating which unit is to be changed.
3. The name of the new unit. This should be enclosed in single quotes and can be at most four characters long.
4. The size of the unit being introduced in terms of the normal unit otherwise used by the program.

Below is a table of the code digits for the units, the quantity to which they apply, and the standard unit used.

Code Digit	Quantity	Standard Reference Unit
1.	horizontal and vertical transverse dimensions	cm
2.	horizontal and vertical angles	mr
3.	vertical ray coordinate (only)	cm
4.	vertical ray angle (only)	mr
6.	momentum spread	% (PC)
8.	length (longitudinal)	meters (M)
9.	magnetic fields	kg
11.	momentum	GeV/c

In certain cases the conversion factor may be omitted and the program will make the required units change by recognizing the symbol used to represent the unit. The automatic units changes available are the same as those in TRANSPORT and the reader is referred to the TRANSPORT manual for a complete listing of them.

MULTIPOLE ABERRATIONS IN QUADRUPOLES AND OTHER  
SPECIAL PARAMETERS -- Type Code 16.0

A number of parameters other than those described so far may be used to give further information about quadrupoles and bending magnets. A parameter introduced on a 16. card applies to all succeeding elements. It may be changed or reset to zero by introducing another 16. type card. Four parameters may be supplied.

1. Type code 16.0.
2. Code digit indicating nature of parameter(s).
3. First special parameter.
4. Second special parameter (if needed).

Below we describe each of the special parameters available. Some of them are the same as those available in the TRANSPORT program and we refer the reader to that manual for further discussion.

Code digit.

- N. A negative code digit indicates a multipole aberration in a quadrupole. Code digit -N indicates a 2N pole. Multipoles up to and including a 40 pole may be included. The first special parameter is equal to  $B_N/ga^{N-1}$ , where  $B_N$  is the pole tip field due to the 2N-pole,  $a$  is the half aperture, and  $g$  is the unaberrated gradient of the quadrupole. The second special parameter is the phase angle

$\alpha_N$  of the multipole. The spatial dependence of the magnetic potential giving rise to a multipole is then given by  $r^N \sin(N\theta - \alpha_N)$ . Since the multipole is normalized to the gradient it may be introduced once and allowed to apply to all succeeding quadrupoles, independent of excitation.

1.  $\epsilon(1)$  - A measure of the second order variation with  $x$  of the magnetic field of a bending magnet. It is equal to the error in field due to this component evaluated at one horizontal unit from the beam axis, divided by the central field value, or  $\Delta B_2/B_0$  at  $x = 1$ . Normally this parameter is set equal to zero. In order for it to produce an effect, a second order calculation must be specified by a 17.0 card. Otherwise, it will be taken to be equal to zero even if a non-zero value is specified.
- 4-5.  $w/2$  and  $g/2$  - The horizontal and vertical apertures respectively of a bending magnet. Both are used in calculating the effect of a fringing field of a bending magnet, but only the vertical gap has an effect in first order. Normally they are taken equal to zero. If either is inserted and

- a 13. 10. card is also inserted, they will act as an aperture stop. A ray will be stopped if it does not go through the aperture specified.
- 7-8.  $k_1$  and  $k_2$  - Values of dimensionless integrals describing the fringing field behavior. Normally  $k_1$ , indicated by index code 7, is taken to be 0.5, and has a first order effect. The quantity  $k_2$ , indicated by index code 8, is normally zero, and is felt only in second order.
- 12-13.  $1/R_1$  and  $1/R_2$  - The reciprocals of the radii of curvature of the entrance and exit faces respectively of a bending magnet. A positive value indicates the curvature is convex. Normally they are taken to be zero, meaning that the faces of the magnet are flat. They have an effect only if a second order card, 17.0, is inserted.
100. Allows specification of a quadrupole aperture other than that specified on the 5. type code card. This aperture may be made elliptical. The two parameters give the horizontal and vertical semi-major axes of this ellipse on the appropriate units. If a circular aperture is desired both quantities must be inserted but are set equal. All quadrupoles before the first 16. 100. card have their apertures taken

from the quadrupole card. All those after any 16. 100. card take their apertures from the last 16. 100. card preceding the quadrupole card. This aperture is observed only if a 13. 10. card is inserted in the deck. The gradient of a quadrupole is always computed from the aperture given on the 5. code card.

SECOND ORDER GEOMETRIC AND MAGNETIC EFFECTS --  
Type Code 17.0

The insertion of a type code 17.0 indicates that second order geometric and magnetic field effects are taken into consideration. All chromatic effects discussed are taken into account whenever  $\Delta p/p$  is not zero. Also, all quadrupole multipole moments are effective whenever specified. A 17. card is necessary to cause second order geometric transfer matrix elements or second order field variations in a bending magnet to have an effect. It is also necessary in order for the magnetic field of a sextupole to have an effect. Without a 17. card a sextupole is taken to be a drift space.

SEXTUPOLE --- Type Code 18.0

A sextupole may be inserted for correction of second order aberrations. Unless a 17. card is included it acts like a drift space. Four parameters are needed.

1. Type code 18.
2. The effective length of the field (normal unit is meters).
3. The pole tip field  $B_0$  (normal unit is kg).
4. The pole tip half aperture  $a$ . The effective strength of the sextupole is  $gB_0 l/a^2$ .

The first and second order transformation elements for a sextupole are given in SLAC-75<sup>2</sup>. In TURTLE these elements are evaluated using the actual momentum of the ray. Therefore, TURTLE can give a very good indication of how effectively chromatic aberrations can be eliminated.

BEAM ROTATION -- Type Code 20.0

A rotation of the coordinates of a ray at a given point in the beam line about the beam axis may be effected by a 20.0 type code. Thus one can simulate a magnet oriented differently than is normally available, or a known rotational misalignment of an element about the beam axis. There are two parameters.

1. Type code 20.0.
2. The angle of rotation in degrees.

The rotation is taken in a clockwise sense about the beam axis.

HISTOGRAMS - Type Codes 50., 51., 52.

A one or two-dimensional histogram of any of the beam coordinates at any point in the beam. A one-dimensional histogram is indicated by a 50. card. The horizontal coordinate of a two-dimensional histogram is indicated by a 51. card. The vertical coordinate of a two-dimensional histogram is indicated by a 52. card. The histogram card is placed at the point in the beam line where one wishes to histogram the ray coordinate specified. Any of these three type cards requires five parameters.

1. Type code 50., 51., or 52. indicating the type of histogram desired.
2. Ray coordinate to be histogrammed, may be 1-4, or 6. Coordinate No. 5 has no significance in TURTLE.
3. Lower limit of the histogram.
4. Upper limit of the histogram.
5. Interval of the histogram. (step size)

A one-dimensional histogram will be created at any point in a beam line where a 50. type code card is inserted. The intervals of the histogram are arranged vertically and specified on the left side of the page. The beam intensity in a given interval is indicated by a horizontal row of the letter "x".

A two-dimensional histogram is actually initiated by the 52. card, while the 51. card merely supplies a value for the horizontal coordinate. Thus the 51. card and the 52. card for a two-dimensional histogram need not occur at the same location in the beam line. However, for each 52. card there must be one 51. card somewhere in the deck preceeding it. Several 52. cards can use the same 51. card to supply the horizontal coordinate for the histogram. The 51. card must simply occur at some point in the deck earlier than the first 52. card. Whenever a 52. card appears, indicating a vertical coordinate and initiating a histogram, the horizontal coordinate is then from the 51. card most recently preceeding it.

For both one- and two-dimensional histograms the coordinates histogrammed with their units and position in the beam line are given below the histogram. For the two-dimensional histogram the coordinated displayed horizontally is given first. Examples of both one- and two-dimensional histograms are given below.

For a two-dimensional histogram the number of rays falling in a given bin is printed directly for 1-9 rays, represented by a letter A-Z for 10-35 rays, and represented by a \$ for more than 35 rays. The \$ was chosen because it is a fairly dark symbol. Sums of the rows and columns are given along the edges of the histogram. The numbers giving the sums of columns are themselves to be read

vertically. Overflow in all four directions is given below the histogram.

A total of 100 histograms in a given beam is allowed. The number of intervals allowed per histogram is limited directly only for the horizontal coordinate of a two-dimensional histogram, the limit being 100. If the limits and interval size specified are such that more than 100 intervals would result, the program readjusts the upper limit so that the number of intervals equals 100. In addition, there is a limit of 10,000 total locations for histogram storage. If  $N$  is the number of intervals used in a one-dimensional histogram, the number of storage locations used is  $N + 2$ . If  $N_A$  and  $N_D$  are the number of intervals used for the horizontal and vertical coordinates respectively of a two-dimensional histogram, the number of locations used is  $(N_A + 1) (N_D + 1) + 5$ .

REFERENCES

1. Karl L. Brown, Sam K. Howry, SLAC Report No. 91 (1970).
2. Karl L. Brown, SLAC Report No. 75 (1969).
3. S. Penner, Rev. Sci. Instr. 32, No. 2, 150-160 (1961).
4. J. K. Cobb, A. W. Burfine, D. R. Jensen, Proceedings of the International Conference on Magnet Technology, 1967, p. 247.

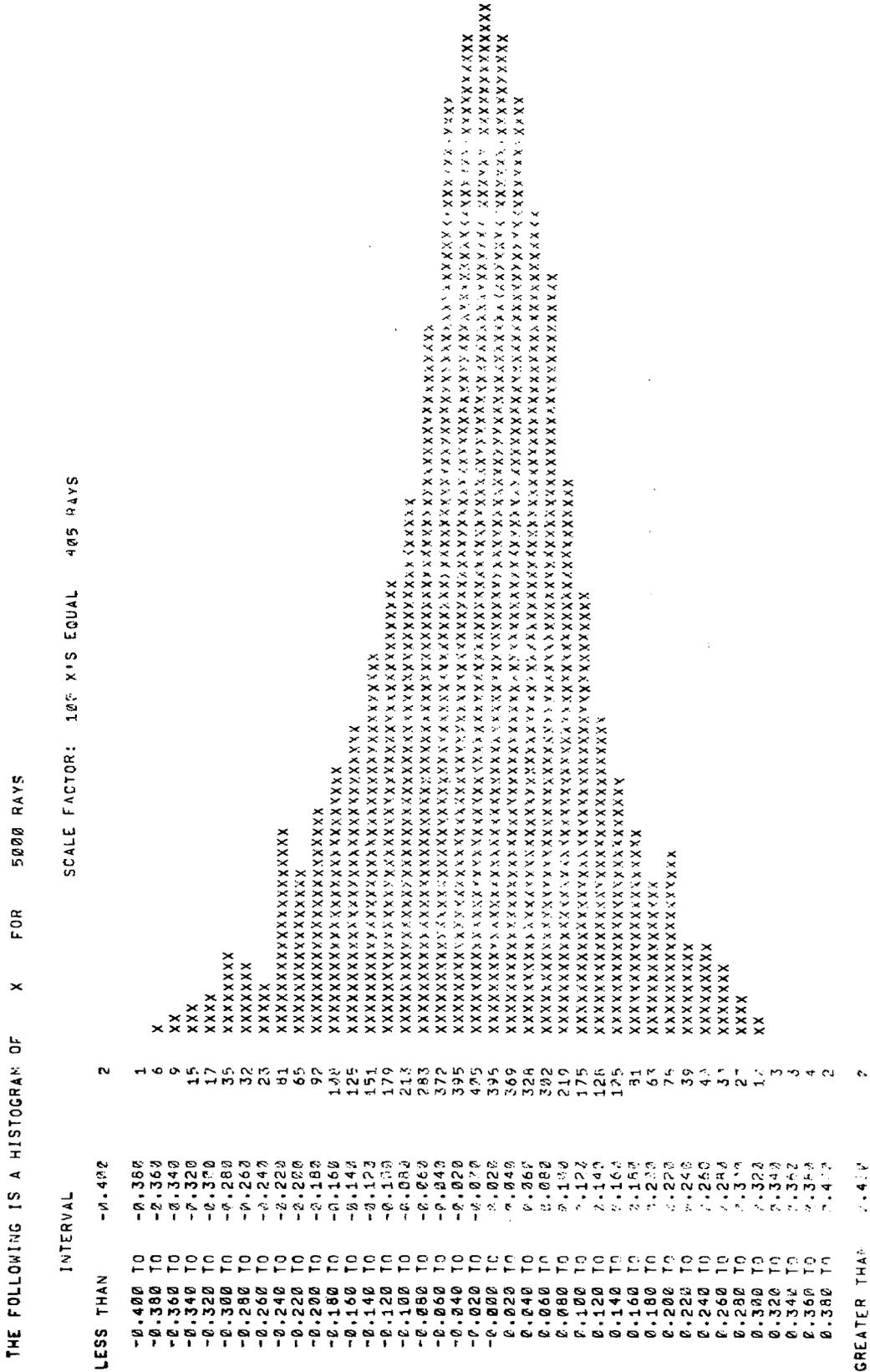


Figure 1 - Example of a one-dimensional histogram.

TWO DIMENSIONAL PLOT OF DP/P VS X		-1.000	-0.800	-0.600	-0.400	-0.200	-0.000	0.200	0.400	0.600	0.800	1.000	TOTALS
-0.380	TO	1	1										6
-0.360	TO	1	1										9
-0.340	TO	1	1										15
-0.320	TO	1	1										17
-0.300	TO	1	1										35
-0.280	TO	1	1										32
-0.260	TO	1	1										23
-0.240	TO	1	1										61
-0.220	TO	1	1										65
-0.200	TO	1	1										92
-0.180	TO	1	1										108
-0.160	TO	1	1										125
-0.140	TO	1	1										151
-0.120	TO	1	1										179
-0.100	TO	1	1										213
-0.080	TO	1	1										283
-0.060	TO	1	1										372
-0.040	TO	1	1										395
-0.020	TO	1	1										405
-0.000	TO	1	1										369
0.020	TO	1	1										328
0.040	TO	1	1										302
0.060	TO	1	1										219
0.080	TO	1	1										175
0.100	TO	1	1										128
0.120	TO	1	1										105
0.140	TO	1	1										81
0.160	TO	1	1										63
0.180	TO	1	1										75
0.200	TO	1	1										39
0.240	TO	1	1										40
0.260	TO	1	1										30
0.280	TO	1	1										20
0.300	TO	1	1										10
0.320	TO	1	1										3
0.340	TO	1	1										3
0.360	TO	1	1										4
TOTALS		178544455857397265	72533942827593768056681178284290550107819967321389845278309035041969246771999864										4995

NO 2 TWO DIMENSIONAL PLOT OF DP/P IN PC 0.000 FT FROM THE TARGET  
X IN CM 1470.000 FT FROM THE TARGET

TOTAL NUMBER OF ENTRIES = 5070 INCLUDING UNDERFLOW AND OVERFLOW AS FOLLOWS

ACROSS	UNDERFLOW	OVERFLOW
DOWN	3	0

Figure 2 - Example of a two-dimensional histogram. The quantity  $\Delta p/p$  is plotted horizontally and x, the horizontal position of the ray is plotted vertically.

September 18, 1973

TO: External Distribution List

FROM: Publications Office, National Accelerator Laboratory

SUBJECT: Correction for NAL external report, NAL-64 by David C. Carey  
"TURTLE" (Trace Unlimited Rays Through Lumped Elements)  
December 1971

Page 34, line 3 should read as follows:

"multipole is then given by  $r^N \sin(N\theta - \alpha_N)$ . " Please make the  
indicated change from cos to sin.