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FIELD QUALITY OF DIPOLES FOR THE DOUBLER*

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I. Introduction

In order to avoid a possible misunderstanding, it must be emphasized at the outset that this note is never meant to be a document to criticize the field quality of doubler dipoles that have been built so far. Anyone with a fair mind will agree that the doubler group has made a remarkable progress in the art of mass production of long superconducting magnets. The heroic effort of the magnet measurement group must also be mentioned. They can now measure all multipoles up to the 30-pole component, body as well as both ends, as a part of the standard measurement procedures. This should be compared with the fact that, of more than one thousand magnets in the main ring, there is only one (four-foot quadrupole) for which we know anything about the multipole fields.

The purpose of this note is simply to tell the fact about the field quality of doubler dipoles that have been built and measured. More specifically, it is a summary of the nine most recent dipoles. Aside from a legitimate question of whether nine is a large enough number to draw any statistical conclusion, one must realize that modifications are still going on (and will go on for sometime to come) in order to improve the field quality. One should therefore never imagine that the ultimate quality of the doubler would be represented

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* "Double, double, toil and trouble;" - three witches -

by these nine dipoles. It is hoped that this note would be especially useful to those who have been wondering about the field quality of doubler dipoles and who have from time to time expressed their concerns. I am grateful to D. Gross, A. Tanner, A. Tollestrup and M. Wake for providing me with their measurement data and for clarifying many important points.

II. Definition of Multipoles

The right-handed coordinate system (x, y, z) is chosen such that

positive x : directed toward the wall in the tunnel,
positive y : directed upward,
positive z : direction of the beam (normal operation).

The angle θ of the cylindrical coordinate (r, θ, z) is measured anti-clockwise from the positive x-axis. The potential for the static magnetic field is

$$\phi = -\sum_{1}^{\infty} \frac{1}{n} k_n r^n \sin(n\theta + \alpha_n), \tag{1}$$

and

$$B_r = -\partial\phi/\partial r = \Sigma\ k_n r^{n-1} \sin(n\theta + \alpha_n), \tag{2}$$

$$B_\theta = -\frac{1}{r}\partial\phi/\partial\theta = \Sigma\ k_n r^{n-1} \cos(n\theta + \alpha_n). \tag{3}$$

For normal fields, $\alpha_n = 0$ or π while for skew fields, $\alpha_n = \pm\pi/2$. In terms of x and y,

$$\text{normal field}\quad B_x = (k_2\cos\alpha_2)y + 2(k_3\cos\alpha_3)xy + (k_4\cos\alpha_4)(3x^2y - y^3) + 4(k_5\cos\alpha_5)(x^3y - xy^3) + \ldots \tag{4}$$

$$B_y = (k_1\cos\alpha_1) + (k_2\cos\alpha_2)x + (k_3\cos\alpha_3)(x^2 - y^2)$$
$$+ (k_4\cos\alpha_4)(x^3 - 3xy^2)$$
$$+ (k_5\cos\alpha_5)(x^4 - 6x^2y^2 + y^4) + \ldots. \quad (5)$$

skew field
$$B_x = (k_1\sin\alpha_1) + (k_2\sin\alpha_2)x + (k_3\sin\alpha_2)(x^2 - y^2)$$
$$+ \ldots.. \quad (6)$$

$$B_y = -(k_2\sin\alpha_2)y - 2(k_3\sin\alpha_3)xy$$
$$-(k_4\sin\alpha_4)(3x^2y - y^3) - \ldots.. \quad (7)$$

From Eq. (5), which is the most familiar, one can identify that

$n = 1$	$k_1\cos\alpha_1 = B_o$	normal dipole
$n = 2$	$k_2\cos\alpha_2 = B_o'$	quadrupole
$n = 3$	$k_3\cos\alpha_3 = B_o''/2$	sextupole
.		
.	$k_n\cos\alpha_n = B_o^{(n-1)}/(n-1)!$	2n-pole

Similarly, from Eq. (6), one can identify various skew multipoles. The direction of dipole component can be measured and this enables us to redefine the positive direction of y-axis such that

$$\alpha_1 = 0.$$

By definition, there is no skew dipole field in the magnet.
The harmonic coefficients b_n and a_n used by the magnet measurement group are defined as

normal: $$b_n = (\int k_n\cos\alpha_n \, d\ell)/(\int k_1 \, d\ell) \quad (8)$$

skew: $$a_n = (\int k_n \sin\alpha_n \, d\ell)/(\int k_1 \, d\ell) \qquad (9)$$

Integrals are over the length of the measuring coil and the standard one is eight feet. The dimension of both b_n and a_n is

$$1/(\text{length})^{n-1}$$

Measurement data on b_n and a_n (after dipole #101) are stored on PDP-10 as the file DIPOLE.DAT[103,122] with cm used as the unit of length. More information on this file is available from Alvin Tanner who is in charge of updating the file as the new data become available. For each magnet, measurements are made at three positions of the coil, one at the center and two at ends. For end measurements, the coil usually extends approximately one foot beyond the end of the magnet lamination. Since the lamination length is 247", three measurements cover the entire length of magnet. In order to find the combined body-end effects, one must take a proper average of three values. For this, it is necessary to know the relative value of $\int k_1 \, d\ell$ for each measurement. This information is also given in the file together with the length between the lamination end and the coil end for two end measurements. There is another file, HA.DAT[103,122], which gives b_n and a_n for magnets #52 to #99. However, this is only for the center field. Except for a few, there were no end field measurements.

III. Summary of Data

Nine dipoles used are
RDA101, RDA102, RDA103, REA104, REA105, RFA107, RHA111, RDC113, RDD114. (#111 to #114 are slightly different from the rest.)

A. Transfer Function (NMR measurement)

The standard current is 2,000A. The unit used is G/A.

#101	9.984	#102	9.989	#103	9.987
#104	9.986	#105	9.976	#107	9.981
#111	9.979	#113	9.989	#114	9.987

For twenty-three samples between #52 and #99, the transfer function is 9.969 with 0.014 (rms).

B. The Vertical Plane Angle

A positive value means the field is pointing toward the wall in the tunnel. The resolution of the stepping motor used for the measurement corresponds to 0.018°.

#101	500A	1,000A	3,000A	
	0.254°	0.257°	0.283°	
#102	500A	1,000A	2,000A	4,000A
	0.209	0.218	0.219	0.208
#103	500A	1,000A	3,000A	
	-.164	-.177	-.179	
#104	500A	1,000A	3,000A	
	0.25	0.25	0.25	
#105	500A	1,000A	3,500A	
	0.054	0.056	0.044	
#107	500A	1,000A	3,000A	
	0.123	0.121	0.123	
#111	500A	1,000A	2,000A	4,000A
	-.125	-.109	-.082	-.108
#113	500A	1,000A	2,000A	3,500A
	0.325	0.334	0.331	0.326

#114	500A	1,000A	4,000A
	0.333	0.322	0.322

C. Relative Values of $\int B\, d\ell$ (stretched wire measurement)

These values are percentage differences relative to the 2,000A excitation in each magnet. Unlike main ring dipoles, there is no standard magnet relative to which all magnets are compared.

#101	500A	1,000A	1,500A	2,500A	3,000A	3,500A	
	% −.55	−.26	.05	.06	.08	.10	

#102 data not yet available

#103	500A	1,000A	1,500A	2,500A	3,000A	3,500A	
	% −.12	−.11	−.04	−.02	.01	−.14	
	+.22	+.03	+.05	−.07	−.13	−.13	

It is not clear if these two sets of measurements were made under the identical condition.

#104	500A	1,000A	1,500A	2,500A	3,000A	3,500A	4,000A
	% −.07	.01	.05	−.01	−.01	−.07	−.09
#105	500A	1,000A	1,500A	2,500A	3,000A	3,500A	4,000A
	% .39	.23	.14	.05	−.15	0.	−.07
#107	500A	1,000A	1,500A	2,500A	3,000A	3,500A	4,000A
	% .36	.09	.09	0.	−.07	−.05	−.14
#111	500A	1,000A	1,500A	2,500A	3,000A	3,500A	4,000A
	% −.71	−.14	−.02	.03	.05	.05	0.
#113	500A	1,000A	1,500A	2,500A	3,000A	3,500A	4,000A
	% .34	0.	.05	−.06	−.06	−.09	−.14
	.43	.08	.11	+.02	−.02	−.01	−.07

#114	500A	1,000A	1,500A	2,500A	3,000A	3,500A	4,000A
%	-.28	-.07	-.07	.03	0.	0.	-.05

D. Multipole Components (rotating coil measurement)

If a magnet has the perfect "dipole" symmetry, i. e.,

$$\phi(\pi + \theta) = - \phi(\theta),$$

all k_n's with an even value of n should be zero (no quadrupole, octupole, etc.). It is of course quite possible that, because of the inevitable imperfection of the coil arrangement, one finds such prohibited multipoles in a dipole. Alvin Tollestrup suggested that, although one must expect a few low order multipoles of this nature, it would be very unlikely that high order multipoles exit as well. Rather, it is more natural to assume that these multipoles are detected because of the shifted center of the measuring coil. The field that will be seen by the beam is then different from the one measured. Without additional information, one must take, more or less arbitrarily, a certain natural multipole and claim that the symmetry-breaking multipoles next to that are entirely due to the shifted center. Two procedures have been used in accordance with Tollestrup's suggestion:

1. Use the normal 18-pole component to explain the normal and skew 16-pole components. Ignore all higher multipoles (normal and skew) and recalculate lower multipole components.
2. Use the normal 22-pole component, the normal and skew 20-pole components instead of 18-pole and 16-pole components.

If the assumption is reasonable, these two procedures should give similar results, at least for low order multipoles. Another indirect check of the consistency is to compare the calculated center coordinates at different excitations. If the coordinates of one magnet depend strongly on the excitation current, something is obviously wrong in the assumption.

The multipole components in the central region of dipoles are given in Tables I-1 to I-6 and the overall (center and two ends combined) components are given in Tables II-1 to II-6. The second procedure explained in the previous page has been tried for I = 1,000A to 4,000A so that one can compare with the results obtained by the first procedure. For each excitation, quantities listed are

First group
- 1st column: number of poles (4=quadrupole, 6=sextupole, etc.)
- 2nd column: ΔB_y (nonlinear normal field) on the median plane at x = 1" in Gauss, average over nine magnets.
- 3rd column: rms value of ΔB_y.
- 4th and 5th column: two extreme values of ΔB_y.

Second group: same as the first group; the quantities are B_x (skew field) on the median plane at x = 1".

The normal dipole field for each excitation current is taken to be

10 (G/A) times the nominal current.

In addition, in Tables I-1 to I-6, calculated coordinates of the center are given for each magnet. Finally, Table III lists parameters useful for estimating various effects of nonlinear field. These are based on the lattice reported by T. Collins.

Table I-1

[illegible]	CD	-0.0936	CM	0.1385	CM
[illegible]	CD	-0.0360	CM	0.1219	CM
RDD114	CU	-0.0299	CM	0.2160	CM
REA105	CD	-0.0427	CM	0.0332	CM
REA105	CU	0.0266	CM	0.0271	CM
RHA111	CD	-0.0330	CM	0.1654	CM
RDA103	CD	-0.0129	CM	-0.0525	CM
RFA107	CD	0.0735	CM	0.2574	CM
REA104	CU	-0.0350	CM	0.2630	CM
RDA102	CD	-0.0059	CM	0.0528	CM

NOMINAL CURRENT: 200 AMPS 10 CASES

4	0.1458	0.2577	-0.3566	0.6115
6	-3.8812	1.0093	-5.5138	-2.1794
8	0.1163	0.1799	-0.1021	0.4375
10	1.4216	0.3155	0.8996	1.9277
12	0.0724	0.2133	-0.2395	0.5678
14	0.3667	0.2415	0.0240	0.8487
16	-0.0000	0.0000	-0.0000	0.0000
18	-2.6922	0.3914	-3.2500	-1.6375

4	0.2366	0.4231	-0.3798	0.7940
6	0.1747	0.3652	-0.4718	0.7278
8	-0.0539	0.4253	-0.5497	0.7366
10	-0.0336	0.1252	-0.2678	0.1274
12	0.1008	0.1735	-0.2384	0.3533
14	-0.0464	0.2525	-0.5644	0.3776

Table I-2

RDA101	CD	-0.0851	CM	0.1311	CM
RDC113	CD	-0.0213	CM	0.1362	CM
RDD114	CU	-0.0410	CM	0.2303	CM
REA105	CD	0.0052	CM	0.0493	CM
REA105	CU	-0.0112	CM	0.0546	CM
RHA111	CD	-0.0282	CM	0.1788	CM
RDA103	CD	-0.0173	CM	-0.0079	CM
RFA107	CD	0.0754	CM	0.2493	CM
REA104	CU	-0.0173	CM	0.2536	CM
RDA102	CD	0.0534	CM	0.0710	CM

NOMINAL CURRENT: 500 AMPS 10 CASES

4	0.3143	0.5704	-0.2939	1.7919
6	-2.0134	1.9407	-5.6750	1.3379
8	0.3188	0.3841	-0.2372	1.0308
10	2.2974	0.6496	1.3934	3.3402
12	0.1246	0.3143	-0.3122	0.8053
14	1.4362	0.3714	0.8215	2.0456
16	-0.0000	0.0000	-0.0000	0.0000
18	-6.8621	0.7025	-8.1170	-5.1662

4	0.4378	0.9506	-1.0253	1.5012
6	0.3745	0.5803	-0.6850	1.3156
8	-0.1058	0.9104	-1.1527	1.7822
10	-0.0333	0.1671	-0.2879	0.1997
12	0.0897	0.2535	-0.2892	0.6335
14	-0.0738	0.3145	-0.7232	0.4443

Table I-3

RDA101	CD	-0.0798 CM	0.1282 CM
RDC113	CD	-0.0135 CM	0.1441 CM
RDD114	CU	-0.0431 CM	0.2369 CM
REA105	CD	0.0226 CM	0.0613 CM
REA105	CU	-0.0289 CM	0.0680 CM
RHA111	CD	-0.0302 CM	0.1802 CM
RDA103	CD	-0.0180 CM	0.0110 CM
RFA107	CD	0.0781 CM	0.2446 CM
REA104	CU	-0.0108 CM	0.2493 CM
RDA102	CD	0.0722 CM	0.0743 CM

NOMINAL CURRENT: 1000 AMPS 10 CASES

4	0.6350	1.2137	-0.6679	3.7674
6	0.1144	3.6094	-7.1660	6.4096
8	0.6101	0.6959	-0.5806	1.8828
10	4.6025	1.2088	2.7519	6.6608
12	0.2688	0.4643	-0.4826	1.1467
14	3.2585	0.5410	2.2743	4.0983
16	-0.0000	0.0000	-0.0000	0.0000
18	-13.7188	1.1908	-16.0709	-11.2080

4	0.8592	1.8740	-2.0187	2.8311
6	0.7213	0.9766	-0.9283	2.5001
8	-0.2476	1.7332	-2.3644	3.4017
10	-0.0768	0.2904	-0.5695	0.4364
12	0.0299	0.4696	-0.7233	1.1467
14	-0.1000	0.3340	-0.7693	0.3388

NOMINAL CURRENT: 1000 AMPS 10 CASES

4	0.6607	1.2226	-0.6559	3.7971
6	0.1499	3.6159	-7.1602	6.3765
8	0.7826	0.9824	-0.8491	2.8617
10	4.6390	1.2230	2.7630	6.6325
12	0.4404	0.7396	-0.7315	2.1795
14	3.1809	0.6009	2.2919	4.1384
16	-0.9702	1.5772	-4.7278	0.8218
18	-12.8801	0.4696	-13.4320	-11.8118
20	0.0000	0.0000	-0.0000	0.0000
22	4.6559	0.9662	3.0605	6.1169

4	0.8850	1.9142	-2.0171	3.2367
6	0.7448	1.0800	-0.9095	3.0432
8	-0.1743	1.8554	-2.2483	3.3897
10	-0.0772	0.3458	-0.5693	0.5445
12	0.1193	0.6177	-0.8837	1.1272
14	-0.1552	0.5650	-1.4660	0.7669
16	-0.1185	2.1650	-3.9559	2.7751
18	-0.0590	1.0406	-1.7016	2.1702

Table I-4

[illegible]	CD	-0.0762 CM	0.1253 CM
[illegible]	CD	-0.0102 CM	0.1490 CM
RUU114	CU	-0.0476 CM	0.2404 CM
REA105	CD	0.0329 CM	0.0778 CM
REA105	CU	-0.0390 CM	0.0776 CM
RHA111	CD	-0.0313 CM	0.1830 CM
RDA103	CD	-0.0177 CM	0.0202 CM
RFA107	CD	0.0774 CM	0.2421 CM
REA104	CU	-0.0061 CM	0.2470 CM
RDA102	CD	0.0811 CM	0.0765 CM

NOMINAL CURRENT: 2000 AMPS 10 CASES

4	1.1944	2.4942	-1.5466	7.6088
6	2.8692	6.9441	-11.6395	14.5997
8	1.2042	1.3561	-1.5451	3.4777
10	9.2716	2.3748	5.4946	13.4466
12	0.5523	0.8242	-1.2024	1.8052
14	6.9002	0.8186	5.5261	8.1120
16	-0.0000	0.0000	-0.0000	0.0000
18	-27.4192	2.2037	-31.9443	-23.3252

4	1.8778	3.7150	-3.8707	5.7003
6	1.4001	1.7856	-1.3501	4.8270
8	-0.5384	3.4021	-4.9057	6.6072
10	-0.1302	0.6075	-1.2397	0.8236
12	-0.1045	0.9662	-1.5773	2.2023
14	-0.1337	0.3993	-0.8418	0.4691

NOMINAL CURRENT: 2000 AMPS 10 CASES

4	1.2718	2.5695	-1.5360	7.9105
6	2.9145	6.9498	-11.6427	14.5509
8	1.4304	1.7260	-1.7602	4.8526
10	9.3199	2.3804	5.5090	13.4105
12	0.7672	1.1346	-1.4398	3.1606
14	6.6650	0.8027	5.5678	7.9691
16	-1.2766	2.3566	-6.9267	1.5439
18	-25.7890	0.7768	-26.7965	-24.2727
20	0.0000	0.0000	-0.0000	0.0000
22	9.5646	1.4318	6.8005	11.7384

4	1.9022	3.7613	-3.8390	6.2995
6	1.4238	1.9107	-1.3392	5.4531
8	-0.5744	3.5265	-4.6748	6.5397
10	-0.1263	0.6414	-1.2389	0.8780
12	-0.1646	1.0432	-1.0526	2.0775
14	-0.2194	0.5600	-1.1624	0.7535
16	[illegible].9047	2.2833	-4.1752	3.[illegible]
18	-0.1367	1.4038	-2.2525	2.1[illegible]

Table I-5

RDA101	CD	-0.0763	CM	0.1242	CM
RUC113	CD	-0.0083	CM	0.1505	CM
RDD114	CU	-0.0476	CM	0.2418	CM
REA105	CD	0.0281	CM	0.0791	CM
REA105	CU	-0.0425	CM	0.0830	CM
RHA111	CD	-0.0325	CM	0.1836	CM
RDA103	CD	-0.0175	CM	0.0241	CM
RFA107	CD	0.0733	CM	0.2414	CM
REA104	CU	-0.0037	CM	0.2456	CM
RDA102	CD	0.0835	CM	0.0770	CM

NOMINAL CURRENT: 3000 AMPS 10 CASES

4	1.6940	3.8411	-2.5233	11.5345
6	3.8157	10.2931	-18.0378	21.2009
8	1.7945	2.0094	-2.4363	5.0630
10	13.8979	3.5379	8.3055	20.1753
12	0.7438	1.3279	-2.5316	2.3373
14	10.9101	1.2518	8.8136	12.8239
16	-0.0000	0.0000	-0.0000	0.0000
18	-41.2995	3.2828	-48.1217	-35.6005

4	3.2986	5.4967	-5.2996	9.1494
6	2.0847	2.6259	-1.8107	7.1952
8	-0.8272	5.0923	-7.4695	9.8147
10	-0.1891	0.9853	-1.9072	1.2225
12	-0.2359	1.4862	-2.4984	3.2315
14	-0.1850	0.4876	-0.8912	0.7149

NOMINAL CURRENT: 3000 AMPS 10 CASES

4	1.7804	3.9497	-2.4959	11.9662
6	3.8750	10.3013	-18.0453	21.1461
8	2.0278	2.4934	-2.8322	6.8223
10	13.9557	3.5376	8.3253	20.1364
12	0.9573	1.7342	-3.0147	4.1530
14	10.5594	1.2399	8.9058	12.1402
16	-1.3647	3.4512	-9.1259	2.8779
18	-38.8121	1.1241	-40.4403	-36.9242
20	0.0000	0.0000	0.0000	0.0000
22	14.2688	2.1961	10.4519	17.4128

4	3.3335	5.5565	-5.2907	9.8349
6	2.1003	2.7853	-1.8217	7.9412
8	-0.8937	5.2222	-7.1421	9.6937
10	-0.1995	1.0137	-1.9040	1.3740
12	-0.3379	1.5702	-2.8611	2.9886
14	-0.2936	0.6140	-1.1619	0.7991
16	1.4513	2.9137	-4.6502	5.2198
18	-0.1436	2.0879	-3.0523	3.3971

Table I-6

[illegible]	CD	-0.0758 CM	0.1234 CM
[illegible]	CD	-0.0074 CM	0.1511 CM
[illegible]	CU	-0.0495 CM	0.2427 CM
MEA105	CD	0.0349 CM	0.0772 CM
REA105	CU	-0.0434 CM	0.0849 CM
RHA111	CD	-0.0329 CM	0.1832 CM
RFA107	CD	0.0723 CM	0.2412 CM
REA104	CU	-0.0023 CM	0.2456 CM
RDA102	CD	0.0848 CM	0.0768 CM

NOMINAL CURRENT: 4000 AMPS 9 CASES

4	0.7899	3.0730	-3.7753	6.3638
6	2.1279	14.2052	-26.2826	26.2563
8	1.9425	2.3133	-3.0904	4.2514
10	18.2018	4.9725	10.8687	26.6992
12	0.7927	1.6971	-3.3167	2.6294
14	15.4517	1.6168	12.7843	17.9075
16	-0.0000	0.0000	-0.0000	0.0000
18	-55.5687	4.5252	-64.4076	-48.1382

4	4.3452	7.1424	-6.4682	13.3004
6	1.9829	2.7513	-2.2257	6.7639
8	-1.9955	6.5667	-10.0045	13.0752
10	-0.4771	1.2539	-2.5368	1.6570
12	-0.5026	2.0456	-3.3930	4.1944
14	-0.2411	0.6864	-1.1612	0.8941

NOMINAL CURRENT: 4000 AMPS 9 CASES

4	0.8541	3.1011	-3.7107	6.5687
6	2.1580	14.1924	-26.3208	26.1844
8	2.0824	2.5093	-3.1730	4.9443
10	18.2097	4.9557	10.8944	26.6474
12	0.9264	1.7260	-3.3397	3.3372
14	15.0593	1.5517	12.8851	16.8608
16	-1.0408	2.8865	-5.5692	3.9219
18	-51.8655	1.4572	-54.0976	-49.7690
20	0.0000	0.0000	-0.0000	0.0000
22	19.5243	2.7586	14.1831	23.2148

4	4.3152	7.1040	-6.4359	13.1332
6	1.8892	2.7458	-2.2636	6.6655
8	-2.2144	6.5576	-9.5864	12.8946
10	-0.5476	1.2037	-2.5348	1.6242
12	-0.7683	2.0589	-3.9095	3.7980
14	-0.3079	0.7630	-1.3984	0.9102
[illegible]	[illegible]	2.9003	-3.1250	6.7001
[illegible]	[illegible]	2.7995	-3.0557	[illegible]

Table II-1 Body and End combined

NOMINAL CURRENT: 200 AMPS 9 CASES

4	0.1827	0.2528	−0.1621	0.6403
6	−6.1745	1.0792	−8.1170	−4.6583
8	0.0855	0.1600	−0.1103	0.3620
10	1.1438	0.3152	0.6722	1.5901
12	0.0353	0.1530	−0.1856	0.3508
14	0.2740	0.1456	0.0996	0.6160
16	−0.0000	0.0000	−0.0000	0.0000
18	−2.7507	0.2790	−3.1856	−2.1698
4	0.0715	0.5100	−0.6527	1.2361
6	0.0233	0.1643	−0.1645	0.3572
8	0.0027	0.4108	−0.4336	0.7949
10	−0.0351	0.1547	−0.3594	0.1455
12	0.0717	0.1745	−0.3296	0.2364
14	−0.0893	0.1549	−0.4508	0.1152

Table II-2

NOMINAL CURRENT: 500 AMPS 9 CASES

4	0.3375	0.6208	−0.8115	1.3508
6	−7.9023	1.9411	−11.3217	−4.5836
8	0.2411	0.3154	−0.2405	0.8016
10	1.6278	0.6533	0.6759	2.5502
12	0.0944	0.2176	−0.2792	0.4891
14	1.2868	0.2778	0.9288	1.7769
16	−0.0000	0.0000	−0.0000	−0.0000
18	−6.9251	0.4996	−7.8646	−5.9262
4	0.0719	1.1263	−1.2409	2.6110
6	0.0753	0.3291	−0.4088	0.7271
8	−0.0456	0.9226	−0.9801	1.8783
10	−0.0705	0.2232	−0.5227	0.1908
12	0.0667	0.2563	−0.3513	0.4849
14	−0.1218	0.2014	−0.5207	0.1834

			normal Sext. only
−1"	23.03	.890	6.22
−.5"	3.08	.016	3.11
.5"	−2.65	.030	−3.11
1"	−21.83	−.468	−6.22
$\frac{\partial B_y}{\partial x}$	G/cm	$\frac{\partial B_x}{\partial x}$	

Table II-3

NOMINAL CURRENT: 1000 AMPS 9 CASES

[illegible]	0.6203	1.3135	-1.9258	2.6858
6	-11.7132	3.5341	-18.4607	-5.8625
8	0.4797	0.5057	-0.3648	1.3787
10	3.2459	1.2438	1.3462	5.1333
12	0.2302	0.3138	-0.3651	0.7852
14	3.0349	0.4635	2.4195	3.6901
16	-0.0000	0.0000	-0.0000	0.0000
18	-13.7771	0.8422	-15.6005	-12.2264

4	0.1225	2.1649	-2.3098	4.8948
6	0.1581	0.5840	-0.7695	1.2518
8	-0.1812	1.7850	-2.0141	3.5410
10	-0.1471	0.3172	-0.6974	0.2599
12	0.0269	0.4429	-0.6178	0.9245
14	-0.1690	0.2650	-0.5622	0.2175

NOMINAL CURRENT: 1000 AMPS 9 CASES

4	0.7052	1.2112	-1.5907	2.3978
6	-11.7061	3.5386	-18.4814	-5.8866
8	0.5268	0.6180	-0.4046	1.7532
10	3.2575	1.2424	1.3541	5.0992
12	0.2259	0.4231	-0.3410	1.1946
14	2.9581	0.4361	2.4293	3.6049
16	-0.1008	1.0966	-2.6952	1.2791
18	-13.1291	0.5373	-14.1547	-12.2930
20	0.0000	0.0000	-0.0000	0.0000
22	4.7968	0.4017	3.9549	5.4234

4	0.1856	2.0456	-2.3264	4.6434
6	0.1581	0.5964	-0.7532	1.2443
8	-0.2100	1.8141	-1.9976	3.5098
10	-0.1615	0.3024	-0.6642	0.2635
12	-0.0035	0.4810	-0.7631	0.9058
14	-0.1464	0.3391	-0.8095	0.3435
16	0.4477	0.9549	-1.6702	1.8583
18	-0.0905	0.3328	-0.4840	0.4902

20.63	1.81	9.22
4.49	.012	4.61
-3.58	.0547	-4.61
-18.50	.29	-9.22

Table II-4

NOMINAL CURRENT: 2000 AMPS 9 CASES

4	1.1742	2.6975	-4.1689	5.3202
6	-20.7783	6.8251	-34.3892	-9.6214
8	0.9619	0.8843	-0.6280	2.4843
10	6.5141	2.4249	2.7619	10.3314
12	0.5074	0.5089	-0.4950	1.3347
14	6.6005	0.8821	5.5408	7.9399
16	-0.0000	0.0000	-0.0000	0.0000
18	-27.5214	1.5790	-31.1760	-24.8390

4	0.3706	4.2126	-4.4950	9.5761
6	0.3146	1.1097	-1.4956	2.3214
8	-0.4724	3.5189	-4.1381	6.8594
10	-0.2825	0.5022	-0.9915	0.4024
12	-0.0481	0.8862	-1.3828	1.8451
14	-0.2517	0.3952	-0.9793	0.3849

NOMINAL CURRENT: 2000 AMPS 9 CASES

4	1.3171	2.5712	-3.6245	5.0438
6	-20.7673	6.8275	-34.4034	-9.6728
8	1.0207	1.0434	-0.6895	3.0076
10	6.5284	2.4171	2.7710	10.2780
12	0.5002	0.6597	-0.5165	1.9301
14	6.4296	0.7844	5.5190	7.4350
16	-0.0710	1.6323	-3.7291	2.1978
18	-26.1974	0.8767	-27.5150	-24.8053
20	0.0000	0.0000	-0.0000	0.0000
22	9.6363	0.6357	8.4224	10.6350

4	0.5025	4.0887	-4.4422	9.4644
6	0.3096	1.1240	-1.4695	2.3286
8	-0.5721	3.5606	-4.0379	6.7894
10	-0.2959	0.4795	-0.9597	0.4075
12	-0.1759	0.9083	-1.6957	1.7190
14	-0.2592	0.3851	-0.8423	0.3293
16	1.2425	1.0525	-0.2679	3.1582
18	-0.1819	0.5257	-0.9271	0.7020

32.97	4.01	16.35
7.87	.02	8.18
-6.11	.10	-8.18
-32.44	1.19	-16.35

Table II-5

NOMINAL CURRENT: 3000 AMPS 9 CASES

4	1.7309	4.1205	-6.5044	7.988[illegible]
6	-31.7440	10.1209	-52.1732	-15.4275
8	1.4612	1.2785	-0.9143	3.6206
10	9.7721	3.6250	4.1908	15.5859
12	0.7505	0.7087	-0.5884	1.8186
14	10.4611	1.3658	8.8320	12.6637
16	-0.0000	0.0000	-0.0000	0.0000
18	-41.4689	2.4598	-47.4092	-37.6134
4	0.9616	6.1817	-6.2261	14.5134
6	0.4518	1.6306	-2.2392	3.3645
8	-0.7747	5.2594	-6.3532	10.1587
10	-0.4204	0.7043	-1.4214	0.5474
12	-0.1253	1.3642	-2.1891	2.7296
14	-0.3438	0.5544	-1.4672	0.5679

NOMINAL CURRENT: 3000 AMPS 9 CASES

4	1.9469	3.9765	-5.7351	7.6786
6	-31.7273	10.1214	-52.2003	-15.5043
8	1.5202	1.4812	-0.9942	4.2766
10	9.7972	3.6091	4.2035	15.5111
12	0.7186	0.9207	-0.6486	2.6186
14	10.2053	1.2235	8.6671	11.8637
16	0.0529	2.2339	-4.6441	3.3016
18	-39.4346	1.2959	-41.3206	-37.5289
20	0.0000	0.0000	-0.0000	0.0000
22	14.4347	1.0269	12.2981	16.0418
4	1.1667	6.0507	-6.1057	14.5000
6	0.4386	1.6554	-2.2079	3.3835
8	-0.9117	5.3111	-6.2148	10.0452
10	-0.4405	0.6725	-1.4257	0.5552
12	-0.3155	1.3790	-2.6725	2.4657
14	-0.3680	0.5072	-1.2756	0.4307
16	1.8505	1.4697	-0.3238	4.3196
18	-0.2680	0.7998	-1.3645	1.1048

-1"	56.94	5.92
-.5"	11.97	.179
.5"	-9.36	.283
1"	-48.70	1.80

$\frac{\partial B_y}{\partial x}$, $\frac{\partial B_x}{\partial x}$ at $y = 0$

Sext.
24.98
12.49
-12.49
-24.98

Table II-6

NOMINAL CURRENT: 4000 AMPS 8 CASES

4	1.4063	5.1209	-9.0113	6.8417
6	-45.1984	13.8387	-71.6202	-22.5268
8	1.6261	1.4275	-1.1688	3.5104
10	12.6798	5.1272	5.4160	20.5842
12	0.8458	0.8151	-0.6779	2.2736
14	14.9312	1.7998	12.3481	17.4039
16	-0.0000	0.0000	-0.0000	0.0000
18	-55.3340	3.1034	-62.3146	-50.6719
4	2.6756	8.4390	-7.3972	19.8646
6	0.4776	2.2569	-2.9219	4.3363
8	-1.9367	6.8872	-8.3834	13.5003
10	-0.4206	0.8613	-1.8860	0.7009
12	-0.2591	1.9030	-2.9535	3.5345
14	-0.4196	0.7421	-1.9023	0.7057

NOMINAL CURRENT: 4000 AMPS 8 CASES

4	1.7739	5.0400	-7.9665	7.2858
6	-45.2155	13.8190	-71.6673	-22.6273
8	1.6224	1.5482	-1.2550	3.7511
10	12.6753	5.1004	5.4311	20.4909
12	0.6850	0.8504	-0.7970	2.1667
14	14.5980	1.5991	12.1267	16.7219
16	0.7382	1.9524	-2.7553	4.3760
18	-52.4717	1.4173	-54.4751	-50.5142
20	0.0000	0.0000	-0.0000	0.0000
22	19.2646	1.2748	16.8213	21.1070
4	2.8847	8.3099	-7.1900	19.9345
6	0.4180	2.2805	-2.8897	4.3664
8	-2.1289	6.9474	-8.2261	13.3546
10	-0.4570	0.8278	-1.8930	0.7079
12	-0.5241	1.9214	-3.6237	3.1506
14	-0.4485	0.6553	-1.6429	0.5314
16	2.5467	2.0065	-0.5021	5.6509
18	-0.2285	1.0987	-1.8094	1.3465

	Normal	skew	Sext (normal) only
-1	76.58	6.78	+35.60
-.5	16.67	.52	+17.80
.5	-14.08	.59	-17.80
1	-64.58	2.44	-35.60

G/cm

Table III Quantities useful for various estimates of the effect of nonlinear field

Lattice by Tom Collins, TM-797 (June 7, 1978)

$\int \beta_H^{\ell} \beta_V^{m} \eta^{n} ds$ over QF, QD, B1 and B2

The unit is (meters).

ℓ	m	n	QF	QD	B1	B2
1	0	1	.7104(E5)	.1394(E5)	.5838(E6)	.2608(E6)
0	1	1	.2258	.4157	.3159	.4656
2	0	2	.3037(E8)	.1168(E7)	.1707(E9)	.3229(E8)
0	2	2	.2913(E7)	.9928	.4827(E8)	.9846
1	0	2	.3035(E6)	.3253(E5)	.2227(E7)	.7603(E6)
0	1	2	.9186(E5)	.1000(E6)	.1181	.1329(E7)
1	0	3	.1460(E7)	.8366(E5)	.9507(E7)	.2459(E7)
0	1	3	.4305(E6)	.2655(E6)	.4941	.4215
1	0	0	.1963(E5)	.6711(E4)	.1909(E6)	.1095(E6)
0	1	0	.6711(E4)	.1964(E5)	.1055	.2004
.5	.5	0	.1130(E5)	.1130(E5)	.1404	.1465
2	0	0	.1963(E7)	.2589(E6)	.1444(E8)	.4571(E7)
0	2	0	.2589(E6)	.1964(E7)	.4435(E7)	.1525(E8)
1	1	0	.6604(E6)	.6605(E6)	.7674(E7)	.7992(E7)
3	0	0	.1972(E9)	.1175(E8)	.1114(E10)	.1954(E9)
1	2	0	.2491(E8)	.6566	.3167(E9)	.5959
4	0	0	.1991 (11)	.6255 (9)	.8729 (11)	.8529 (10)