Fermilab

UPC 012

October 12, 1978

To: Alvin Tollestrup

From: S. Ohnuma S.O.

It has been pointed out to me by Helen that harmonic components in my files (DCH1.DAT, etc.) are somewhat different from your values. Before we go too far, I thought I should clarify my procedures so that, if there are any deficiencies, I should be able to correct them. Please read the attached note carefully. RDA101 is used as the example to explain the procedures I have been using.

cc: H. Edwards
M. Harrison

A. <u>Averaging</u>* (RDA101)

Center (C): coil 96", $V_C = 3175.3$

Downstream End (D): coil 96", 7" out of the lamination, $V_D = 2936.9$

Upstream End (U): coil 96", 16" out of the lamination, $V_U = 2761.1$

Since the lamination length is 247", there will be no overlapping if the coil length for the center measurement is 78",

$$96" + 96" + 78" = 270" = 247" + 7" + 16".$$

We assume that there is no variation (longitudinally) in the center area. If the coil length is 78" instead of 96", the voltage will be

$$V_C' = V_C \times (78"/96") = 2579.9$$

All harmonic coefficients are of course unchanged. The average value of a harmonic coefficient b_n has been calculated from the expression

$$(b_n)_{av} = \frac{2579.9\ (b_n)_C + 2936.9\ (b_n)_D + 2761.1\ (b_n)_U}{2579.9 + 2936.9 + 2761.1}$$

For the normal and skew 30-pole components, the shift of the center does not give any change.

normal: at 1,000A
$$b_C = 0.78090\times10^{-10}/\text{cm}^{14}$$
$$b_D = 0.68705\times10^{-10}/\text{cm}^{14}$$
$$b_U = 0.42024\times10^{-10}/\text{cm}^{14}$$

$$b_{av} = 0.62732\times10^{-10}/\text{cm}^{14} = \underline{0.29185}\times10^{-4}/\text{inch}^{14}$$

The underlined value is in my file DCH1.DAT

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* The averaging procedure has been modified. See p. 5, Appendix I. Coil length is 94", not 96".

skew: at 4,000A

$$a_C = -\ 0.47087\times10^{-10}/\text{cm}^{14}$$
$$a_D = -\ 0.10550\times10^{-10}/\text{cm}^{14}$$
$$a_U = -\ 0.24277\times10^{-10}/\text{cm}^{14}$$

$$a_{av} = -\ 0.26517\times10^{-10}/\text{cm}^{14} = \underline{-\ 0.12336}\times10^{-4}/\text{inch}^{14}$$

B. Finding the Center

The normal 18-pole component is large in the design. As you suggested, we assume that the normal and skew 16-pole components are mostly due to the normal 18-pole component with the shifted center. Once the center position is determined this way, we recalculate all harmonic components with the center adjustment. The normal and skew 16-pole components are then not zero but they should be very small if our assumption is justified. The center coordinates should not depend on the excitation current except for 200A and possibly for 500A. We could use the center coordinates evaluated at 4,000A for all currents. However, in preparing files, I have used the center coordinates evaluated at each current. The dependence of coordinates on the excitation current is indeed insignificant.

Notations:

$$B_r = \Sigma\ k_n r^n \cos\alpha_n\ \sin(n+1)\theta \qquad \text{(normal)}$$
$$+\ \Sigma\ k_n r^n \sin\alpha_n\ \cos(n+1)\theta \qquad \text{(skew)}$$
$$B_\theta = \Sigma\ k_n r^n \cos\alpha_n\ \cos(n+1)\theta \qquad \text{(normal)}$$
$$-\ \Sigma\ k_n r^n \sin\alpha_n\ \sin(n+1)\theta \qquad \text{(skew)}$$

$n = 0$ dipole, $n = 1$ quadrupole, etc.

$$B_x = B_r \cos\theta - B_\theta \sin\theta$$

$$B_y = B_r \sin\theta + B_\theta \cos\theta$$

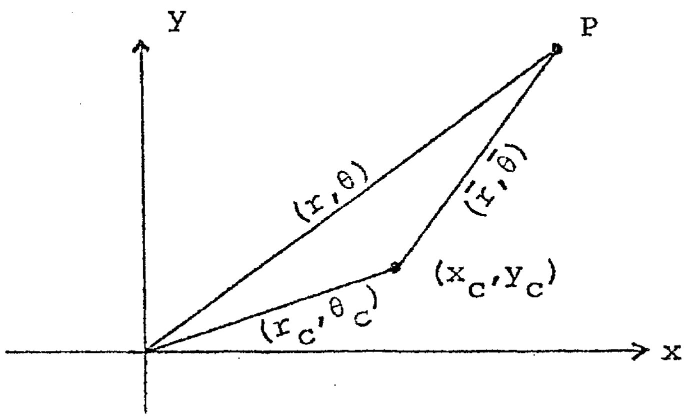
By definition, $\alpha_o = 0$.

harmonic coefficients: normal $b_n \equiv (k_n \cos\alpha_n)/k_o$

skew $a_n \equiv (k_n \sin\alpha_n)/k_o$

dipole field $B_o \equiv k_o$

$$(B_y - B_o)/B_o = \Sigma\, b_n r^n \cos(n\theta) - \Sigma\, a_n r^n \sin(n\theta)$$



center of the measurement

$x = y = 0$

center of the cold bore

$x = x_c, \quad y = y_c$

$$(B_y - B_o)/B_o = \mathrm{Re}\, \Sigma b_n (re^{i\theta})^n - \mathrm{Im}\, \Sigma a_n (re^{i\theta})^n$$
$$= \mathrm{Re}\, \Sigma \bar{b}_n (\bar{r}e^{i\bar{\theta}})^n - \mathrm{Im}\, \Sigma \bar{a}_n (\bar{r}e^{i\bar{\theta}})^n$$

(b_n, a_n): measured (off-center) multipoles

$(\bar{b}_n, \bar{a}_n)$: real multipoles

$$(re^{i\theta})^n = (\bar{r}e^{i\bar{\theta}} + r_c e^{i\theta_c})^n$$
$$= \sum_{\ell}^{n} \left[{n \atop \ell}\right] (r_c e^{i\theta_c})^{n-\ell} (\bar{r}e^{i\bar{\theta}})^{\ell}$$

$\left[{n \atop \ell}\right]$ = binomial coefficients

$$\bar{b}_\ell = \sum_n b_n \left[{n \atop \ell}\right] r_c^{\,n-\ell} \cos(n-\ell)\,\theta_c - \sum_n a_n \left[{n \atop \ell}\right] r_c^{\,n-\ell} \sin(n-\ell)\,\theta_c$$

$$\bar{a}_\ell = \sum_n b_n \left[{n \atop \ell}\right] r_c^{\,n-\ell} \sin(n-\ell)\,\theta_c + \sum_n a_n \left[{n \atop \ell}\right] r_c^{\,n-\ell} \cos(n-\ell)\,\theta_c$$

* In order to find $x_c = r_c \cos\theta_c$ and $y_c = r_c \sin\theta_c$, we take b_8, b_7 and a_7 and make $\bar{b}_7 = \bar{a}_7 = 0$;

$$\bar{b}_7 = b_7 + 8 \times b_8\, x_c = 0, \quad \bar{a}_7 = a_7 + 8 \times b_8\, y_c = 0,$$

that is,

$$x_c = -\, b_7/(8 \times b_8) \quad \text{and} \quad y_c = -\, a_7/(8 \times b_8).$$

Finally, find r_c and θ_c from (x_c, y_c) to calculate $\bar{b}_\ell$ and $\bar{a}_\ell$ using two expressions given above.

example: RDA101, Center at 1,000A

$$b_8 = -\ 0.78591 \times 10^{-6}/\text{cm}^8$$
$$b_7 = -\ 0.50150 \times 10^{-6}/\text{cm}^7$$
$$a_7 = +\ 0.80609 \times 10^{-6}/\text{cm}^7$$

$$x_c = -\ 0.0798 \text{ cm} = -\ 31.40 \text{ mil}$$
$$y_c = +\ 0.1282 \text{ cm} = +\ 50.48 \text{ mil}$$

At other excitation currents, we find

	x_c	y_c
500A	-.0851cm	.1311cm
4,000A	-.0758	.1234

The change from 500A to 4,000A is less than 5 mil.

* The procedure to find (x_c, y_c) has been modified. See p.5, Appendix II.

Appendix I

For downstream and upstream measurements, the length of coil sticking out of the lamination is not always measured accurately. It is felt that one should probably use 252" as the effective length of the dipole field. The variation of this from magnet to magnet should certainly be less that 0.5". The average value of b_n is

$$(b_n)_{av} = \frac{V_D(b_n)_D + V_U(b_n)_U + (V_C' - V_D - V_U)(b_n)_C}{V_C'}$$

where $V_C' = V_C\ (252''/94'') = 2.681\ V_C$

The coil length is 94" and <u>not</u> 96" which was the value used previously. All files have been revised using this procedure for averaging.

Appendix II

A. Tollestrup suggested that one should probably take a_8 together with b_8 to compute the center coordinates.

$$\bar{b}_7 = b_7 + 8\ b_8 x_c - 8\ a_8 y_c = 0$$

$$\bar{a}_7 = a_7 + 8\ a_8 x_c + 8\ b_8 y_c = 0$$

and

$$x_c = -\frac{a_7 a_8 + b_7 b_8}{8(a_8^2 + b_8^2)}, \quad y_c = \frac{b_7 a_8 - a_7 b_8}{8(a_8^2 + b_8^2)}$$