# RF REQUIREMENTS FOR TEVATRON USE WITH $\overline{P}P$ COLLIDING BEAMS

## Accelerating Cavity Spacing

When the Tevatron (T'tron) is to be used only for unilateral acceleration of protons (in either direction) the location and spacing of rf cavities is not dictated by any consideration other than available space. For simultaneous bilateral acceleration and subsequent storage of protons and anti-protons, however, some restrictions must be imposed upon cavity spacing. By appropriate spacing and phasing of the rf fields in individual cavities some selected aspects of bilateral operation may be optimized. The requirements for $\overline{p}p$ operation are listed as follows.

1. The rf system must create sufficient phase space (bucket area) for simultaneous bilateral acceleration and storage of protons and anti-protons. Because the total charge and longitudinal emittance of protons and anti-protons will almost certainly be quite different, the required bucket areas will not necessarily be the same.

2. The rf system should provide the capability for moving the bunch collision point azimuthally over some reasonable range.

3. The system may be required to allow for independent control of the phase and amplitude (bucket location and size) of the proton and anti-proton buckets.

If requirement three is satisfied, requirement two is automatically satisfied also, but requirement two may be satisfied with a system that does not necessarily satisfy requirement three.

In this note a spacing scheme is presented which can provide for all of the above requirements. It is assumed that T'tron cavities will be sufficiently short so that they can be placed with their "effective gap" locations one-half wavelength apart. The "effective gap" location of a cavity is assumed to be at its center regardless of internal cavity structure.

As a basic unit two adjacent cavities are placed such that their effective gaps are three quarter wavelengths apart. A particle moving "downstream" arrives at the second gap at a time phase $3\pi/2$ radians later than its arrival at the upstream gap. If the downstream cavity gap voltage leads the upstream voltage by $\pi/2$ radians, a particle moving downstream will see the gap voltages exactly in phase (modulo $2\pi$ radians), and consequently the two cavities provide their maximum bucket area for such a particle. A particle moving upstream, however, will see the gap voltages exactly out of phase and, if the gap voltages are equal, such a particle will see no net voltage. An additional similarly spaced doublet, with opposite relative phasing, can be placed arbitrarily close to the first pair. There is good reason to space the gaps of the nearest neighbors of adjacent doublets one-half wavelength apart so a pair of doublets (four cavities) will occupy a space of approximately two and one-half wavelengths. All of the listed requirements can be satisfied through the use of such doublets. Upstream and downstream doublets may be driven from separate rf sources and operated at different amplitudes and phases.

As an example, Figure 1 shows three doublets, the two outside doublets providing downstream bucket area while the center doublet provides upstream bucket area. A simple fanout system is shown to demonstrate that the required phasing can be accomplished using easily available components, quadruture hybrid junctions and $\pi$ radian splitters. In the array shown, with all gap voltages equal, the downstream bucket area (probably for protons) will be larger than the upstream area by a factor 1.414 during beam storage. During acceleration the bucket area difference will be slightly larger because the upstream particles will require a larger synchronous phase angle and consequently the "moving bucket factor" reduction will be larger.

The cavity spacing described is essentially a series of cavities (1, 4 and 5) with their gaps spaced an integral number of half-wavelengths apart and another group (2, 3 and 6) with the same relative spacing but all displaced by one-quarter wavelength. Such an array of cavities can be phased in a slightly different manner to provide a greater total bilateral bucket area if requirement three is relinquished. Such a phasing scheme is shown in Figure 2. The upstream and downstream bucket areas are equal and each effective voltage is 0.707 $V_{total}$.

With the phasing shown in Figure 2 the intersection point is one-eighth wavelength to the left of the mid-point of the array. If the voltages of cavities 2, 3, and 6 are reduced to zero the intersection point will move to the mid-point of the array (with slight reduction in bucket area) and if cavities 2, 3, and 6 are raised to maximum voltage with opposite phase the intersection point will move to a point one-eigth wavelength to the right of the mid-point.

It is possible that the phasing of Figure 2 could be used during acceleration and the phasing switched, after storage energy is reached, to the phasing of Figure 1 to provide orthogonal control.

Because of the quite different geometry of the two fan-out systems this phase switching would be difficult and would require great care to avoid phase space dilution or loss of particles.

Bucket Areas and Required Voltages

In the previous section cavity spacings were described which could provide bilateral bucket areas, but the number of cavities could not be specified because it remains unclear how much voltage each cavity will deliver. It is possible, however, to make some estimate of the required voltages based on bunch lengths and expected longitudinal emittance. A reasonable estimate for the proton beam would be 12 bunches, each containing $10^{11}$ protons with longitudinal emittance per bunch of 2 eV-Sec., and total bunch length of 5 nSec. At 1 TeV these parameters dictate a voltage requirement of $1.95 \times 10^{6}$ volts resulting in a stationary bucket area of 15 eV-Sec. The anti-proton beam is expected to consist of 12 bunches of 2 nSec. width, each containing $10^{10}$ anti-protons with a longitudinal emittance of 0.08 eV-Sec. This would require a stationary bucket area of 3.6 eV Sec. and a ring voltage of 120 KV.

The voltage requirement for protons may be in excess of that which is easily obtainable with the added burden of generating the orthogonal anti-proton voltage. It is possible that the high harmonic damping cavities in the main ring will allow the longitudinal emittance to remain at a lower value than 2.0 eV Sec. so that the required proton bunch length may be obtained with somewhat less voltage.

## Bunch Reconfiguration in the Main Ring

Colliding beam $\bar{p}p$ physics in the T'tron will, of course, require that reconfigured bunches be injected from the main ring, and this will entail some modification of the beam handling equipment in the main ring. One or more low harmonic number cavities will be required. Voltage and frequency requirements for such cavities may be inferred from the following re-bunching scenario.

Recent storage studies in the main ring have indicated that, at intensity $2 \times 10^{11}$ protons (in about 1066 of 1113 available buckets) about 90 percent of the beam is contained within bunches 3 nSec. in length and such bunches appear to be matched to stationary buckets created by 1.25 MV ring voltage at the beginning of a store.

Using cannonical phase space coordinates W = R/h dP and $\phi_{rf} = \Theta h$ the half-angle of such bunches is

$$\phi_b = \pi\text{(bunch length)(rf frequency)} = 0.50 \text{ radians.}$$

The bucket "height" $W_{max}$, at 100 GeV is

$$W_{max} = f(E_s, \gamma_t, R) \left[\frac{V}{h^3}\right]^{\frac{1}{2}} = 16 \left[\frac{V}{h^3}\right]^{\frac{1}{2}} = 4.3 \times 10^{-4} V^{\frac{1}{2}}$$

where h - 1113.

If V = $1.25 \times 10^6$, $W_{max}$ = 0.48 eV Sec.

The momentum excursion of a bunch within such a bucket is

$$W_b = W_{max} \text{ Sin } \frac{\phi}{2} = (0.48)(\text{Sin } 0.25) = 0.12 \text{ eV Sec.}$$

The bunch area becomes,

$$S_{bunch} = \pi\phi W_b = 0.188 \text{ eV Sec.}$$

If the rf voltage is reduced adiabatically until the bucket area equals the bunch area (about 3 kV) then turned off, a phase space dilution of about a factor of $\pi/2$ will occur resulting in a debunched longitudinal emittance of about 0.295 eV Sec per bucket. Treating a single rf period this emittance can be expressed $S = \Delta E \Delta T_{rf}$ or

$$\Delta E = \frac{S}{\Delta T_{rf}} = Sf_{rf} = (0.295)(53.102 \times 10^6) = 15.6 \times 10^6 \text{ eV.}$$

The number of protons in each bucket was originally

$$N = \frac{(0.9)(2 \times 10^{13})}{1066} = 1.69 \times 10^{10} \text{ protons}$$

so a density can be ascribed to the debunched beam.

$$\rho = \frac{1.69 \times 10^{10}}{0.295} = 5.7 \times 10^{10} \text{ Protons/eV Sec.}$$

We intend to create bunches containing $10^{11}$ protons each so we can now state that, with no further dilution, we must recapture into each h = 1113 bucket a charge bunch occupying a phase space of

$$S_{bunch} = \frac{10^{11}}{5.7 \times 10^{10}} = 1.75 \text{ eV Sec.}$$

In order to compensate for subsequent beam loss during extraction, injection into the doubler, and acceleration, we will examine the requirements for reconfiguration and recapture of a charge phase space area of 2 eV Sec. Recapture is to be accomplished using a lower harmonic number cavity by relocating charge in an appropriate way, then quickly turning on the h = 1113 rf system with correct bucket area to enclose the required phase space. The voltage required to create a bucket at h=1113 of 2 eV Sec is;

$$\text{Area} = 8W_{max} = 34.5 \times 10^{-4}\ V^{\frac{1}{2}} = 2$$

whence $V_{rf} = 336$ kV.

This voltage requirement is easily achieved and sufficient over-voltage is available so that acceleration of such a bunch with adequate bucket area is not a problem.

At capture, because the bucket is full, the maximum energy spread of the captured beam is obtained by examining the bucket height.

$$W_{max} = \frac{A}{8} = 0.25 \text{ eV Sec.}$$

$$dp = \frac{h}{R} W_{max} = 0.278$$

$$dE \simeq d(cp) = (0.278)(3 \times 10^8) = 83 \times 10^6 \text{ eV}$$

The energy (momentum) spread of this beam is $\pm 0.83 \times 10^{-3}$ which is well within the observed useful momentum aperture of the main ring at 100 GeV.

The requirements on the relocating cavity are now clear. It must coalesce the charge from regions adjacent to particular h = 1113 bucket centers while increasing the energy spread from ± 7.8 meV to ± 83 meV. The relocated charge should remain stationary with respect to the centers of the h = 1113 buckets which implies that the relocating cavity should operate on a harmonic number which is one of the integral factors of 1113.

Consider a relocating cavity with harmonic number h = 21.

$$f = \frac{(21)(53.1025 \times 10^6)}{1113} = 1.0019 \text{ mHz.}$$

Each bucket created by such a cavity will contain phase space equivalent to 53 of the original buckets. Consider a "bunch" within such a bucket which extends over half of the bucket phase length, $-\pi/2$ to $\pi/2$. Such a "bunch" will contain about 26 of the original phase space areas. If this region of phase space within the h=21 bucket is allowed to undergo one quarter of a phase oscillation the charge within this region will rotate into roughly a vertical strip at the bucket center. (The remaining charge will make a long "tail" out to the unstable fixed points.) The momentum limits of this strip are required to be $\pm$ 0.278 eV.Sec/Meter, whence

$$W_b = \frac{R}{h}\,dp = \frac{(10^3)(0.278)}{21} = 13.24 \text{ eV Sec.}$$

$$W_{max} = \frac{W_b}{\sin\frac{\pi}{4}} = 18.72 \text{ eV Sec.} = 16\left|\frac{V}{h^3}\right|^{\frac{1}{2}}$$

$$V = 21^3\left|\frac{18.72}{16}\right|^2 = 12.7 \text{ kV.}$$

This is a manageable voltage for a 1 mHz cavity.

The emittance within the rotated strip is approximately

$$S_{strip} \simeq (26)(0.293) = 7.62 \text{ eV Sec}$$

so $$\Delta T = \frac{7.62}{(2)(83 \times 10^6)} = 46 \times 10^{-9} \text{ Sec.}$$

The synchrotron period for h = 21 and the required voltage of 12.7 kV is about 0.6 seconds so the rotation process will require about 150 MSec. At this time the vertical strip encompasses about two and one-half h = 1113 rf periods with the correct energy spread, so even if the strip is not perfectly alligned vertically the h = 1113 bucket should easily enclose a dense region of phase space and re-capture the required charge.

If the same exercise is done using a cavity at h = 53, f = 2.53 mHz,each bucket contains 21 of the original phase spaces.

The required voltage is 32 kV and the rotated strip width, containing about 2.93 eV Sec is 17.7 nSec in length. Because this length is slightly shorter than a h = 1113 period the allignment problem appears more difficult and that, together with the larger voltage requirement, makes the h = 53 option appear less acceptable than h = 21.

The question of whether the main ring beam can be debunched at 100 GeV without instability has been studied in a recent storage study. Main ring beam was debunched at 100 GeV in a manner close to that described above and was observed to have a beam storage lifetime consistent with that observed for bunched beam. Attempts to measure the momentum spread of the debunched beam by Schottky scan techniques were not successful possibly due to insufficient detector sensitivity and/or magnet ripple. Further attempts to make such measurements are intended.

Actual study and verification of the parameters described above cannot at present be accomplished because we do not have an rf cavity of sufficiently large voltage which can operate in the main ring below 5 MHz. Reconfiguration studies at frequencies which are not integral sub-harmonics of 1113 are nevertheless useful should be pursued. To this end a surplus low frequency cavity from the PPA accelerator has been installed in the main ring and various bunch reconfiguration experiments will be done in coming months to clarify the precise requirements.

f = 53.1045 MHz
$\lambda$ = 5.647 M.

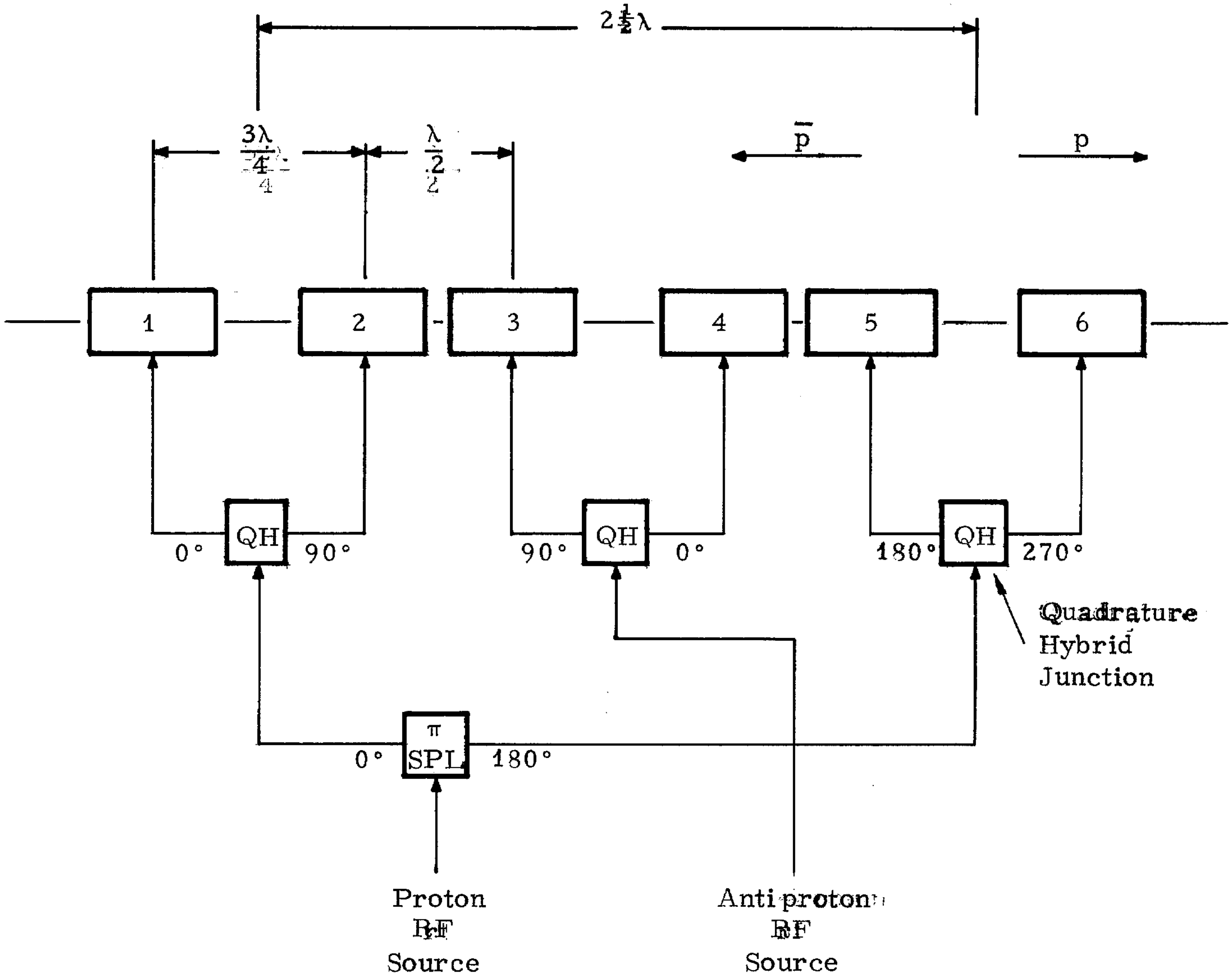


Fig. 1. Tevatron rf cavity spacing for simultaneous bilateral orthogonal treatment of protons and anti-protons.

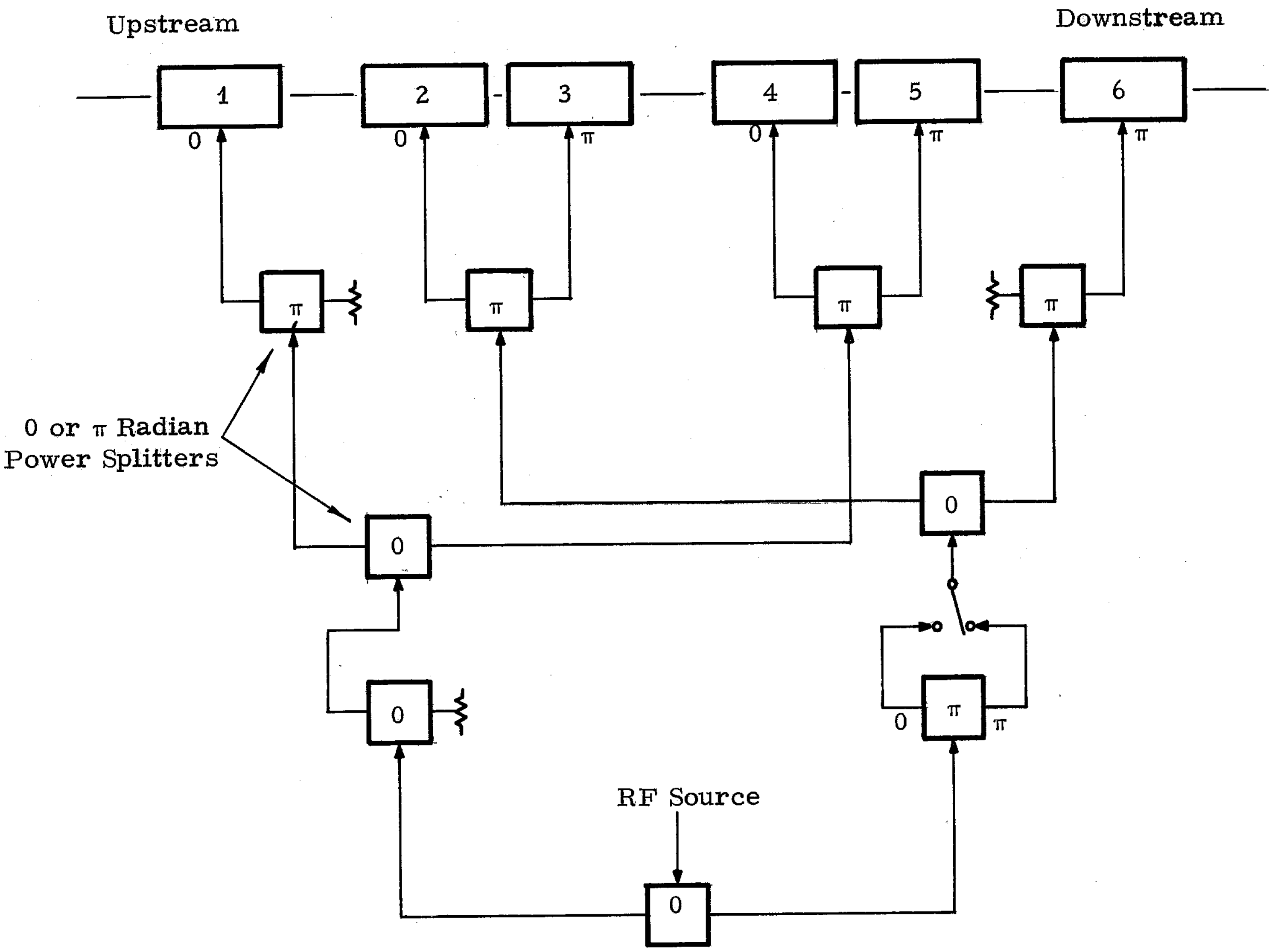


Fig. 2. Bilateral cavity phasing scheme yielding more bucket area but not orthogonal control of $\overline{p}$ and p bucket parameters.