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Progress Report on the Study of the Effects
Which are Intensity Dependent in the
Energy Doubler

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The present note represents the progress report of a theoretical study to estimate the behavior of the beam in the Energy Doubler under the effects of its own intensity.

The questions that with this study we try to answer are the following:

Is the beam unstable against coherent oscillations?

If so, what are the thresholds and growth times of the instabilities?

How are they related quantitatively to the items in the vacuum chamber?

What is the parametric dependence of the instabilities with beam energy and intensity and the main machine parameters?

Could the instabilities cause beam losses?

What are the effects on the beam size?

In case of bad effects of these instabilities, how can one avoid them or cure them?

Are there associated other side effects like, for instance, beam power losses?

This study requires two parallel, simultaneous efforts. One deals with the electromagnetic problem, namely with the calculations of the fields produced by the beam interacting with the elements of the vacuum chamber, the other with the beam dynamics to determine the effects of these fields on the beam itself.

In the first kind of efforts one practically solves the Maxwell equations with assigned boundary conditions which represent the vacuum chamber elements themselves. The fields induced by the beam are then characterized by the notion of coupling impedance. For the second kind of problems one basically solves equations of motion with specified fields, external and those induced by the beam intensity. The equations of motion are solved with assigned initial conditions (spreads). The solution, here, leads to the notion of instability threshold and growth time, and eventually of final beam size (overshoot).

All this sounds like a logical approach and it is indeed what was undertaken during the past two decades or so of accelerator physics. This approach is the only valid one; actually it is the only one available to us. In principle, provided that someone can perform all the required calculations, it predicts exactly what the beam behavior should be. In practice, the amount of calculations involved is immense and very difficult if not impossible. Approximations are then required, and a theoretical model for determining the solution of the problem is nothing else but a formulation of these approximation. If some formulae fail to agree with an actual beam observation, this is not because the general approach is wrong but, probably, because those formulae were derived according to a theoretical model based on approximations which do not apply to the particular situation. It is up to the experimental physicist to find out when an approximation does not apply and point it out to the theorist.

For our study of the problem applied to the Energy Doubler, we shall rely on those theoretical models that have been worked out

in the past and that are by now commonly used in accelerator design and operation all around the world. We are all aware of the approximations used in these theories, but we shall refrain ourselves to produce better approximated theories, because so far there has not been any valid justification to provide for any designing of a new machine like the Energy Doubler. All the theories available give a fair description of the beam behavior in all the accelerators of the world, including the Fermilab complex.

Our study is made of two parts:

- A. Behavior of the beam in the longitudinal plane (synchrotron motion).
- B. Behavior of the beam in the transverse plane (betatron motion) and, eventually, its coupling to the longitudinal plane.

So far we have investigated only problems associated with part A, and actually we still have to conclude this part. The following subjects have already been investigated more or less in great detail:

1. Estimation of the Coupling Impedance
2. Coasting Beam Stability (Longitudinal)
3. Individual Bunch Longitudinal Instabilities (Microwave Instabilities)
4. Bunch-to-Bunch Longitudinal Instabilities
5. Estimate of the Coupling Impedance for the Bellow to very high frequencies.
6. Determination of the Beam Power Losses due to the Resistivity of the Walls.

In the following we review the results of our analysis for

each of these topics.

1. Estimation of the Coupling Impedance

For the low frequency range we have the following estimates:

a. Resistive Wall

Wall resistivity assumed at 4.2° k is

$$\rho = 52 \mu\Omega \times \text{cm}$$

The pipe radius assumed is 4 cm. This leads to the following coupling impedance at the mode number n

$$Z_n = (1-i) 8.1 \sqrt{n} \text{ ohms}$$

b. Bellows, without RF fingers and vacuum chamber extension. For 1000 bellows

$$Z_n = -i 0.3 n \text{ ohms}$$

c. Conductive Plates (Pickups as designed and tested by E. Higgins)

$$Z_n = -i 0.4 n \text{ ohms}$$

for 250 of them.

d. Vacuum Chamber Steps

We assumed 2000 steps with a ratio of the outer radius to the inner radius of 1.2

$$Z_n = -i 0.2 n \text{ ohms}$$

The total contributions of these items, so far is summarized in the following table:

\underline{n}	$ Z_n/n $
1	12.4 ohm
10	4.5
100	2.0
1000	1.3
10000	1.0

We estimate for the Energy Doubler

$$|Z_n/n| \approx 10 \text{ ohm}$$

In the high frequency range we have estimated the contribution of the vacuum chamber wall steps. With the same parameters as shown before we project constant value of 38 kΩ. We have also estimated the contribution of the bellows (see later).

2. Coasting Beam Stability

There might be a need to debunch the beam in the Energy Doubler at 100 GeV. Therefore it is required to check the beam stability for this operation. We use the following formulae for threshold and growth time

$$|Z/n| \approx \frac{E|\eta|}{eI_0} \left(\frac{\Delta p}{p}\right)^2 \quad (1)$$

$$1/\tau \sim n\omega_0 \sqrt{\frac{eI_0|\eta||Z/n|}{2\pi E}} \quad (2)$$

where

I_0 , average beam current = 0.15 A

$\eta = \gamma_t^{-2} - \gamma^2 \approx 0.0028$ for $\gamma \gg \gamma_t$

E , beam energy = 100 GeV

ω_0 , 2π x revolution frequency = 2π x 47.7 kHz

$\Delta p/p$, momentum spread (FWHM)

IF we take $|Z/n| = 10$ ohm we obtain

$$(\Delta p/p)_{\text{threshold}} = 7.2 \times 10^{-5}$$

which corresponds to a longitudinal area per main RF wavelength (~5.6 m)

$$S_{\text{threshold}} = 0.22 \text{ eV}\cdot\text{s}$$

assuming gaussian distribution and including 95% of the beam, and

$$1/\tau \approx 0.022 \text{ n sec}^{-1}$$

3. Microwave Instabilities

We will take a look at three different models predicting individual bunch longitudinal instabilities.

a. Coasting Beam Theory applied to Bunches. We still make use of Eqs. (1) and (2) but we replace I_0 with the peak current value I_p , and the momentum spread corresponds to the center of the bunch. The results for $|Z/n| = 10 \text{ ohm}$ are shown in the following table:

	V = 1 MV		V = 4 MV	
	$N_B = 2 \times 10^{10}$	$N_B = 10^{11}$	$N_B = 2 \times 10^{10}$	$N_B = 10^{11}$
σ	16 cm	27 cm	10 cm	17 cm
δ	0.35×10^{-4}	0.6×10^{-4}	0.5×10^{-4}	0.8×10^{-4}
S	0.4 eV·s	1.2 eV·s	0.3 eV·s	0.9 eV·s
I_p	2.4 A	7.2 A	3.8 A	11.3 A
$n\tau$	24.7 s	14.3 s	19.7 s	11.4 s

where V is the total RF voltage, N_B the number of particles per bunch, σ the bunch rms length, δ the bunch rms energy spread (relative), S the bunch area for 95% of the beam and gaussian distribution

$$S = 6\pi \sigma \delta E / c$$

and τ is the growth time of the instability in absence of Landau damping. The range of interest of the mode numbers here is $n \gg 1000$. The assumed RF is 53.1 MHz.

b. Resistive Theory. This is based on a combination of theoretical analysis and computer simulations. The following threshold for the bunch area is used

$$S_{th} \approx 0.53 \frac{I_b Z_R}{\omega_0} \delta_t \sqrt{\frac{E}{eV}}$$

where I_b is the average current per bunch, and Z_R is a real (positive) impedance which, according to this model, is assumed constant over the entire frequency range. The results are shown in the following table for $Z_R = 100 \text{ k}\Omega$

	V = 1 MV		V = 4 MV	
	$N_B = 2 \times 10^{10}$	$N_B = 10^{11}$	$N_B = 2 \times 10^{11}$	$N_B = 10^{11}$
S	0.5 eV·s	2.5 eV·s	0.25 eV·s	1.3 eV·s
σ	18 cm	39 cm	9 cm	20 cm
δ	4.4×10^{-5}	9.8×10^{-5}	4.4×10^{-5}	9.8×10^{-5}
I_p	2.1 A	4.9 A	4.3 A	9.6 A

The assumed RF is again 53.1 MHz.

c. Low-Q resonator model. In this theory the impedance Z is approximated to that of a low-Q resonating circuit. As for the resistive theory, also in this case the results are obtained from a combination of analytical effort and computer simulation. The stability condition (1) still applies provided that n is intended as the ratio of the resonance frequency to the revolution frequency, $\Delta p/p$ is now replaced by the rms value of the energy spread and

$$Z = \frac{R Z_R}{\sqrt{4\pi} \sigma_M}$$

R, machine radius = 1000 m

M, number of bunches

Z_R , shunt impedance of the low-Q resonator.

The results are shown in the following table for $V = 1$ MV, $n = 2 \times 10^4$ (1 GHz)

$N_B \backslash Z_R$	0.1 M Ω	1.0 M Ω
2×10^{10}	0.18	0.82
10^{11}	0.52	2.40

What is tabulated is the threshold value of the bunch area S in eV·s. Again the RF is 53.1 MHz.

All the results that have been shown in the previous tables apply to a beam of 1000 GeV.

4. Bunch-to-Bunch Longitudinal Instabilities

For our analysis we greatly relied on the work done by F. Sacharer in this field.

We believe that the offending impedance, for these kinds of instabilities, can always be approximated by that of a resonating circuit; there is, though, no limitation on the figure of merit of this circuit. The main result can be expressed as the following

$$\frac{|\Delta\omega_m|}{\omega_s} = \frac{M I_0 R_s F_m D}{2\pi B h V \cos\phi_s}$$

where

ω_s , 2π x unperturbed phase oscillation frequency

$\Delta\omega_m$, complex, angular shift of the frequency of the m-th pole traveling around the contour of the bunch ($\omega_m \sim m\omega_s$)

ϕ_s , synchronous phase angle (for slow acceleration $|\cos\phi_s| \sim 1$)

h, RF harmonic number = 1113

B, bunching factor

$$B = \frac{M\omega}{R\sqrt{2\pi}}$$

R_s , shunt impedance of the resonator.

D depends on the attenuation of the induced signal between bunches, namely on the Q of the resonator. F_m is a form factor that specifies the efficiency with which the resonator can drive a given mode m. We define an equivalent shunt impedance

$$R_s^{eq} = R_s F_m D \leq R_s$$

The stability condition is

$$\left| \frac{\Delta\omega_m}{\omega_s} \right| < \frac{\sqrt{m'}}{2} \frac{\Omega}{\omega_s}$$

where Ω is the rms spread in ω_s across the bunch. Provided that Q is not exceedingly too small, the threshold depends only on the average current I_0 . The main results are shown in the following, if we take

E	1000 GeV
I_0	0,15 A (2×10^{13} ppp)
V	1 MV
h	1113
M	1113
R_s^{eq}	30 k Ω
m	1 (dipole mode)

then

σ	40 cm
S	2.5 eV.s
τ/T_s	39
Ω/ω_s	1.8%

τ/T_s is the ratio of the instability growth time to the synchrotron oscillations period. The instability is reasonably slow, and we

expect that a longitudinal damper of 200 MHz bandwidth and capable of a few kV can provide beam stability. We have also shown the required spread Ω/ω_s for stability in case this has to be supplied by a higher harmonic cavity (Landau cavity). This is also the spread required, eventually, from bunch to bunch, provided the variation is fast enough.

5. Estimate of the Coupling Impedance for the Bellow

In Figs. 1 and 2 we show two possible configurations of the bellow in the Energy Doubler. Fig. 1 shows the proposed configuration. A vacuum chamber extension and an RF finger (not shown in the figure) across the gap AB should shield electromagnetically the bellow from the beam. Unfortunately the RF fingers cause trouble during the installation of the magnet and it is not obvious that they are really "shorting" the gap between A and B. Therefore we analyzed the impedance for the two configurations shown in Figs. 1 and 2.

The main parameters are

l , length	2.75 cm
b , pipe radius	3.94 cm
τ , wiggle amplitude and distance of bellow from the chamber wall	0.635 cm
w , separation of wiggles	0.131 cm

The bellows are made of stainless steel and we assumed the same resistivity of the rest of the vacuum chamber.

The analysis is done by making use of transmission line models. One can identify a host of sharp resonances up to some cutoff which depends on the transverse dimensions of the line.

The results for 1000 bellows are shown in the following table

	Bellows	
	Shielded	Unshielded
k_{\max}	27	10
$\omega_k/2\pi$ (GHz)	$0.878(1+2k)$	$11.8(1+2k)$
R_k	$0.65/\sqrt{1+2k}$	$0.27/\sqrt{1+2k}$
Q_k	$163\sqrt{1+2k}$	$73\sqrt{1+2k}$

k_{\max} is the number of resonances expected up to cutoff. ω_k , R_k and Q_k are respectively ($2\pi \times$) the frequency, the shunt impedance and the figure of merit of the k -th resonance ($k = 0, 1, 2, \dots, k_{\max}$).

Definitively the unshielded bellow would be a better choice: there are fewer resonance lines, in higher frequency range and with lower shunt impedance.

6. Determination of the Beam Power Losses due to the Resistivity of the Wall

For the case of 1113 bunches equally spaced and equally populated, and assuming gaussian distribution of the current within a bunch, the beam power loss per magnet W is shown in Fig. 3. It depends drastically on the average current I_A and on the bunch length. For instance at 1000 GeV and $I_A = 0.15$ A

S	0.1 eV·s	0.4 eV·s
σ	8.0 cm	16 cm
P	20	7.6
W (per magnet)	120 mW	46 mW

Conclusion

Based on the results we have obtained so far, we can provisionally conclude that the stability conditions for the beam against longitudinal coherent oscillations are not too severe. If one can keep the beam bunches always at the threshold of stability, the resulting bunch sizes (length and momentum spread) are quite manageable and can be easily fitted within the Energy Doubler momentum aperture (hopefully better than 10^{-3}) and the RF buckets. A Bunch Spreader, therefore, seems to be very useful and highly desirable. This device probably will not be exactly the equivalent of what we have now in the Main Ring, but a more sophisticated one. One should be able to continuously control with it the bunch size to stay always within the limits of stability. By doing this one would accomplish two things: first, one would avoid unnecessarily growth of the bunch beyond the threshold values (overshoot); second, one would get an even beam which is a requirement for a smooth extraction in slow spill mode. The technical features of the Bunch-Spreader are really not yet known and some effort should be devoted to its design.

The following are the topics we are investigating or in the process of investigating:

- i. Determination of the impedance of a pickup device (Ed Higgins) in the high frequency range.
- ii. Consideration about beam power losses in the Energy Doubler: It is well known that a bunched beam induces to the wall a current rich in harmonics. Such a current causes energy loss when it traverses resistive elements. This effect is very important in electron storage rings (PEP, PETRA) because of the very large

peak currents involved (thousands of amperes), but so far it was passed almost unnoticed in proton machines where the peak currents are considerably lower (amperes). Nevertheless the Energy Doubler superconducting magnets have cold bore and the refrigeration system is capable of absorbing a few watts from each magnet, as a consequence power losses induced by the beam are now important. We have estimated the losses due to the resistivity of the wall, shown before, we have to now estimate the losses also from other devices like bellows and pickups. To do this we have to first estimate the coupling impedance of these objects in a considerably wide frequency range.

After this is done we shall turn our effort to evaluate the possible cures, if cures are required. We mention some:

- a. CEA cavity for the dynamic compensation of energy losses according to the microwave resistive model.
- b. Landau cavity to enhance a synchrotron frequency spread within a bunch.
- c. RF modulation to enhance a synchrotron frequency spread between bunches.
- d. Longitudinal Damper

Finally it remains completely untouched the second part of our studies: the stability of the beam in the transverse plane.

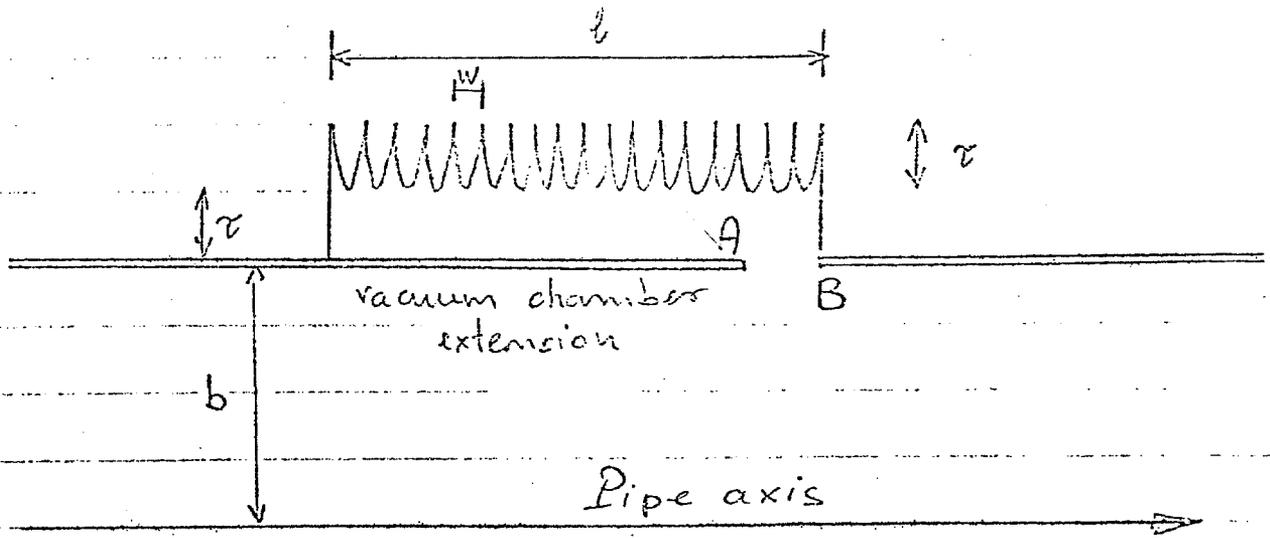


Fig. 1 Shielded Bellow

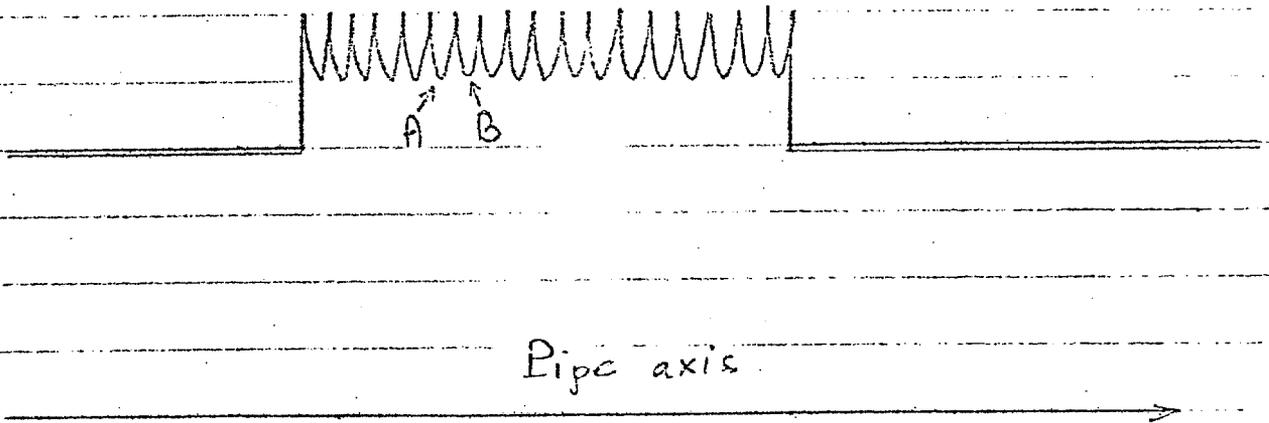


Fig. 2 Un-shielded Bellow

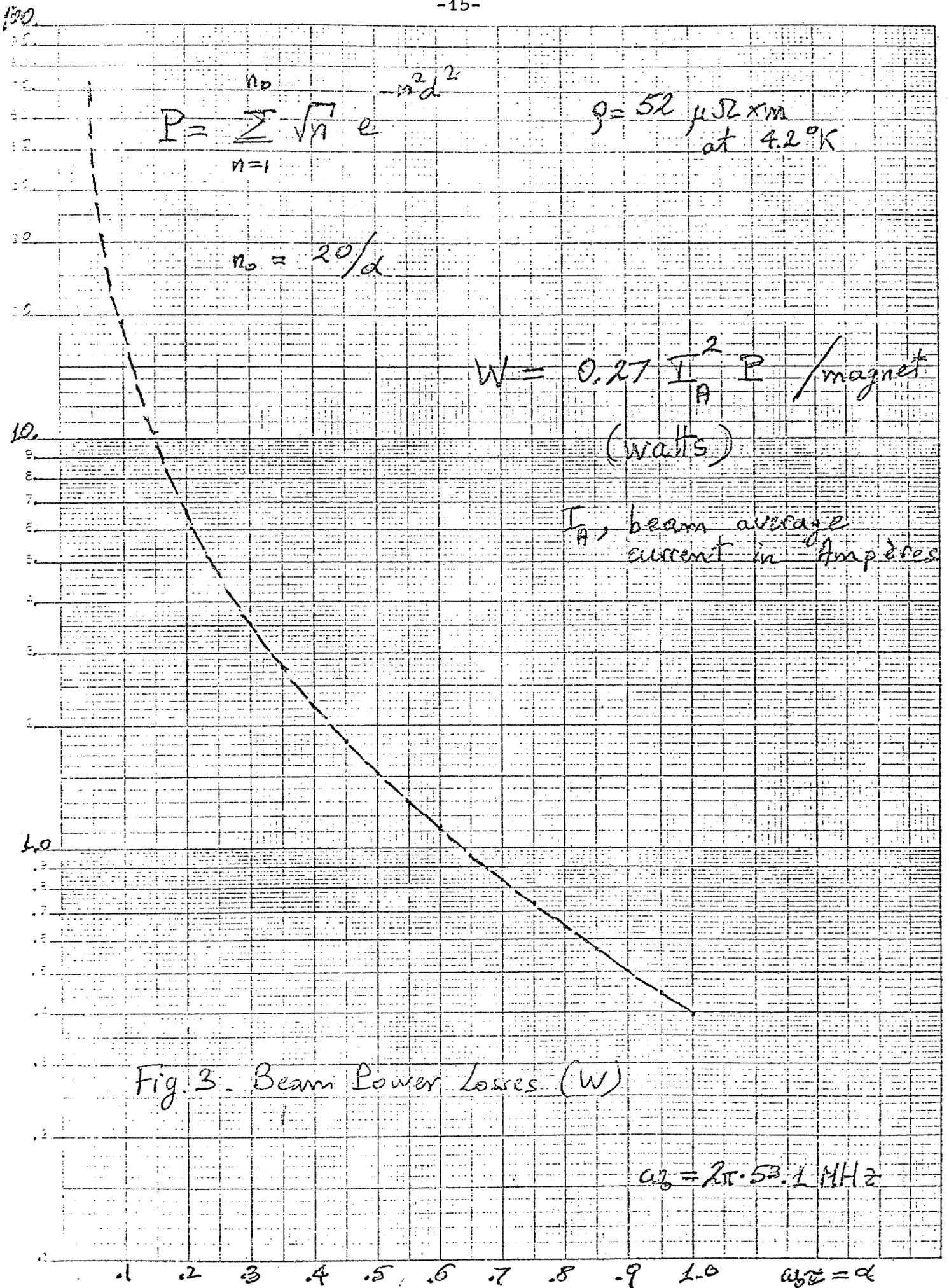


Fig. 3. Beam Power Losses (W)