UPC 081

Bunch-to-Bunch Longitudinal Instabilities

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In this note we review the bunch to bunch longitudinal instabilities for the Energy Doubler. The definition and discussion of the impedance which is the crucial parameter for our analysis can be found in a previous note dealing with a similar subject: the Individual Bunch Longitudinal Instabilities.[10]

1. Definition of Modes of Instability

There are two possible groups of modes of longitudinal instabilities:

a. The internal bunch[2] mode (m). The instability can develop around the contour of a bunch as shown in Fig. 1. The mode number m (=1,2,3...) is the number of waves around the contour of the bunch.

Mode number m = 1 is usually called dipole mode, m = 2 quadrupole, and so on. The wave shown in Fig. 1 propagates around the center at a frequency m times the phase oscillation frequency.

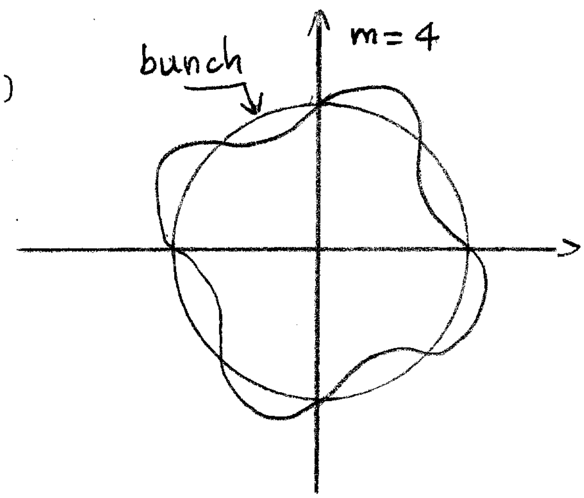

Fig. 1. Internal Bunch Mode

b. Bunch-to-Bunch[1] mode (p). For a given internal-bunch mode number m, the instability can propagate from bunch to bunch as shown in Fig. 2.

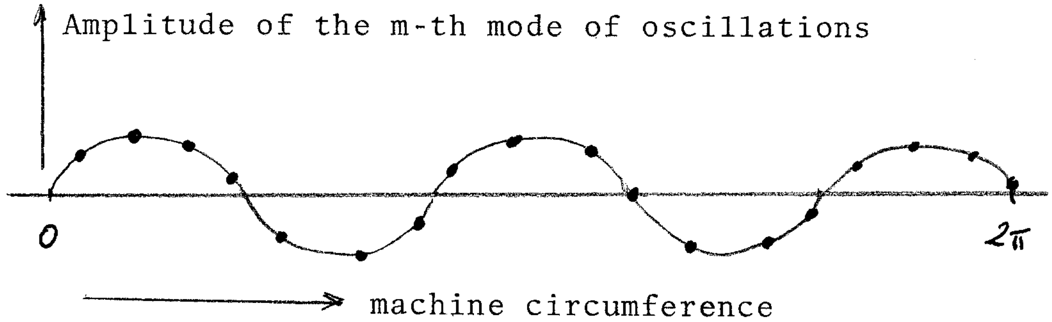

Fig. 2. Bunch-to-Bunch Mode

The number p of waves around the circumference of the machine is the mode number. The instability of one bunch is carried over to the next with a phase slip of

$$2\pi p/M$$

where M is the total number of bunches. The number M is not necessarily equal to the RF harmonic number h, but in the following we shall always make the assumption the M bunches are equally spaced.

2. The Complex Shift[3-5]

A charged bunched beam induces a current at the walls of the vacuum chamber. This current when crossing elements along the vacuum pipe (cavities, bellows, pick-ups,...) creates high-frequency voltages which act back on the beam. The main effect of these voltages is a shift of the phase oscillation frequency modulated within a bunch m-fold and from bunch-to-bunch p-fold.

In the case the bunch has a m-mode perturbation the complex shift of the wave propagation around the contour of the bunch is given by[1]

$$\frac{\Delta\omega_m}{\omega_s} = -i\sqrt{m}\,\frac{I M^2 \omega_0}{2\pi h B^3 V \cos\varphi_s}\;\frac{\sum_k \frac{Z_{\parallel}(\omega_k)}{\omega_k} h_m(\omega_k)}{\sum_k h_m(\omega_k)} \tag{1}$$

where ω_m is about m times the unperturbed synchrotron angular frequency ω_s, I the average current in M bunches, B the bunching factor defined as the ratio of the bunch length to the bunch separation, ϕ_s the stable phase, ω_o the revolution angular frequency, $Z_{/\!/}(\omega_k)$ the longitudinal (complex) coupling impedance[10] at $\omega = \omega_k$ and $h_m(\omega_k)$ the Sacharer's functions[1].

The shift $\Delta\omega_m$ is a function of the two integers: m, the internal bunch mode number, and p, the bunch-bunch mode number. If Q_s is the number of phase oscillations per revolution,

$$\omega_k = (k + m\, Q_s)\, \omega_o$$

with $-\infty<k<+\infty$ for a single bunch (M = 1) or several bunches oscillating independently (namely the impedance is such the wake field resulting at very short range), and $k = p+k_1M$, $-\infty<k_1<+\infty$ for coupled motion of the bunches.

Observe that the growth time of the instability is given by the imaginary part of the r.h. side of (1). Therefore if the impedance $Z_{/\!/}$ is a pure reactance no instability can occur. The real part of the impedance $Z_{/\!/}$ is responsible for the growth of a perturbation, whereas the imaginary part causes simply a shift of the synchrotron frequency at the mode of the perturbation.

In the following we shall consider three different impedance models:

a. Smooth impedance

$$\left|\frac{Z_{/\!/}}{k}\right| = \left|\frac{Z_{/\!/}}{\omega_k/\omega_o}\right| = Z_o \tag{2}$$

where Z_o is related (actually equal) to the longitudinal coupling impedance[10] $|Z/n|$ as defined for coasting beam. Inserting (2) in (1) gives

$$\left|\frac{\Delta\omega_m}{\omega_s}\right| \sim \sqrt{m}\ \frac{I M^2 Z_0}{2\pi B^3 V h \cos\phi_s} \tag{3}$$

b. Sharp (High-Q) Resonance[6-7]

In this case (1) is replaced by

$$\left|\frac{\Delta\omega_m}{\omega_s}\right| = \frac{M I R_s F_m D}{2\pi B h \sqrt{\cos\phi_s}} \tag{4}$$

where R_s is the shunt impedance of the resonator. D depends on the attenuation of induced signal between bunches, namely on the Q of the resonator. At most D = 1. F_m is a form factor that specifies the efficiency with which the resonator can drive a given mode. The maximum value of F_m decreases with m. At most $F_m = 1$, which is the maximum for the dipole mode m = 1. In the following we shall take D = 1, which corresponds to no attenuation ($Q \to \infty$) and $F_m = 1$. Doing so, we overestimate the frequency shift $\Delta\omega_m$.

c. Low-Q Resonator[10]

This would be a better representation of the machine impedance than one given by the model A, which assumes an impedance which increases linearly over the entire frequency range. The low-Q resonator could exhibit a resonance in the GHz range. This model can obviously be represented again by a shift (4), though in this case D is considerably smaller than unity.

3. The Stability Condition[2,3,5]

Let Ω be the rms spread in ω_s across the bunch. The stability condition is

$$\left|\frac{\Delta\omega_m}{\omega_s}\right| < \frac{\sqrt{m}}{2}\ \frac{\Omega}{\omega_s}$$

From the expansion of the nonlinear motion of the phase oscillations we have

$$\frac{\Omega}{\omega_s} = \frac{h^2\sigma^2}{24\, R^2}$$

where σ is the rms bunch length and R the machine radius. Also we have for the bunching factor

$$B = \frac{M\sigma}{R\sqrt{2\pi}}$$

which is valid when we assumed a gaussian distribution of the charge along the bunch, and B is defined as the ratio of the peak current to the average beam current.

Combining everything together then one derives the following conditions on the rms bunch length. From (3)

$$\sigma \gtrsim R\left(48\sqrt{2\pi}\,\frac{I_b Z_o}{V h^3 \cos\phi_s}\right)^{1/5} \tag{5}$$

and from (4)

$$\sigma \gtrsim R\left(24\sqrt{\frac{2}{\pi m}}\,\frac{I\, R_s}{V h^3 \cos\phi_s}\right)^{1/3} \tag{6}$$

In the smooth case, (5), the stability condition depends only on the average current per bunch I_b, in the case of a resonator (6) it depends on the total average current I. In either case there is no dependence on the total number of bunches.

Observe also that whereas in (5) there is no dependence of either mode number p or m, in (6) there is only a dependence on the internal bunch mode number m, and the worst situation corresponds to the dipole mode m = 1. In the following we shall consider therefore only this case.

There is no dependence on the beam energy.

In the case of slow acceleration $\cos\phi_s \sim 1$.

We take V = 1 MV and h = 1113.

The main results are shown in Tables I, II and III for the following two cases:

Case I: 1100 bunches with $2\text{x}10^{10}$ particles per bunch,

$I = 150$ mA and $I_b = 0.15$ mA

Case II: 15 bunches with 10^{11} particles per bunch,

$I = 10$ mA and $I_b = 0.75$ mA

As explained in an another note[10] we shall assume for the Energy Doubler $Z_o = 50\ \Omega$. For the high-Q resonator model it is plausible to assume for the shunt impedance R_s a value around 30 kΩ, because this is certainly larger than that was possible to achieve in the Main Ring cavities with dampers and it is also what could be expected from spurious resonances in other elements around the ring. For the high-Q resonator model very likely $D \sim 1$.

For the low-Q resonator model we model we shall take, in a way arbitraily

$$DR_s \sim 100\ \text{k}\Omega$$

In Tables I, II and III, the growth time of the instability (in units of the phase oscillation period T_s) has been calculated by taking the imaginary part of (1). In the same tables we give the bunch area S for 1000 GeV and 1 MV total RF voltage. The case of the resonator (High-Q, impedance model B, Table II) looks allright, but in the smooth case model (case A, Table I) the bunch area at the threshold exceeds the available bucket area

Table I. Bunch Parameters for the Impedance Model A, $Z_o = 50\ \Omega$

	Case I	Case II
σ(m)	.92	1.27
S(eV·s)	13	25
τ/T_s	7.3	3.8

Table II. Bunch Parameters for the Impedance Model B, $R_s = 30\ k\Omega$, $D \simeq 1$

	Case I	Case II
σ(m)	.40	.16
S(eV·s)	2.5	0.4
τ/T_s	39	234

Table III. Bunch Parameters for the Impedance Model C, $DR_s = 100\ k\Omega$

σ(m)	.60	.24
S(eV·s)	5.6	0.9
τ/T_s	18	105

of 10 eV·s. Observe that things do not improve by raising the RF voltage.

Our estimations for the "smooth case" are certainly too pessimistic, because the impedance model is not very realistic. We know that actually $Z_{/\!/}$ reaches a peak around one or a few GHz and then rolls off. Table III which corresponds to this low-Q model gives likely a better estimate of the beam parameters at the threshold of stability.

Observe that $\tau >> T_s$ and therefore the instability is reasonably slow.

The bunch parameters as shown in either Table II or III are reasonable, namely bunches of that size should be easily fitted within the available momentum aperture of the Energy Doubler.

Finally observe that Case I is more sensitive to the instability than Case II.

4. Possible Cures

We give a list below of possible cures without going, though, into too many technical details.

a. As shown in Table II and III the bunch size at the threshold of stability is reasonably small compared to the available RF bucket area and momentum aperture of the Energy Doubler. Also the momentum spread of the beam is reasonably small.[10] Therefore if one can keep the beam continuously at the stability limit very few problems will arise. To accomplish this one needs a "Bunch Spreader" which continuously, in a dynamical fashion, blows-up the bunches to the desired area value.

b. Reduce Z_0 and the shunt impedance R_s of the spurious modes with a controlled bookkeeping of the various items which are to be installed within the vacuum chamber of the Energy Doubler.

Damping of parassitic modes may end up to be crucial.[1]

c. Enhance a synchrotron frequency spread across the bunch with a higher harmonic number cavity (Landau Cavity)[2,9]. With this technique, at least in principle, one can avoid an increase of the bunch area. For instance in Case I with the low-Q impedance model (C, R_sD = 100 kΩ) a spread

$$\frac{\Omega}{\omega_s} = 1.8\%$$

is required with a fundamental RF voltage of 1 MV (h = 1113).

d. Bunch-to-bunch modes can also be made stable with a spread in synchrotron frequency from bunch-to-bunch.[2] The amounts of requirement for spreads are the same as in the previous case c. Modulation of the RF voltage at the revolution frequency enhances this spread. But it is necessary that the variation of the phase oscillation frequency from bunch-to-bunch is faster than the mode of instability itself.

e. Longitudinal Active Damper.[8] Because the bunches are too short (their length corresponds to GHz frequency range), one can only hope to damp the dipole mode (m = 1), which involves oscillations of the bunches barycentre, The bunch-to-bunch mode n = 0 is already taken care by the low-level phase loop in the RF system. As one can see in Tables II and III, the instabilities are expected to grow relatively slowly (one second) therefore a slow, wideband longitudinal damper should be feasible.

References

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