UPC 089

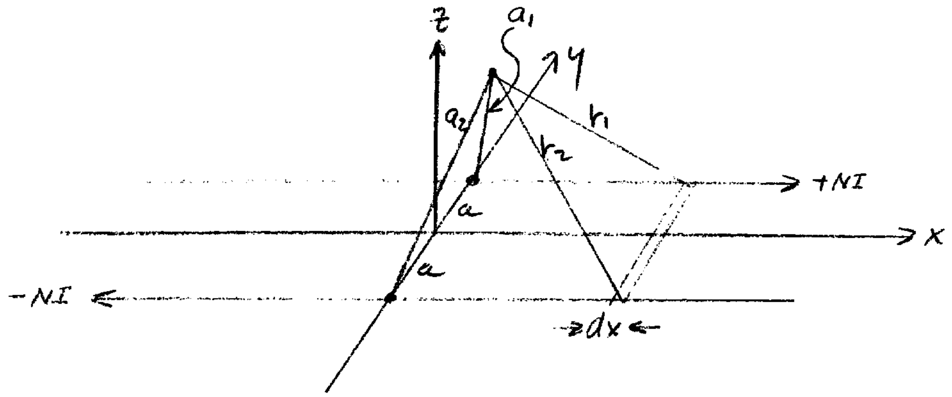
Fermilab

Shafer
3/29/79

Coil Size Scaling Laws for AC Losses in ED Dipoles

The purpose of this note is to estimate how the AC losses in ED dipoles scale when the coil diameter is changed. As the major AC losses now occur in the cryostat assembly, (See UPC #31) which is outside the ED coil, the AC losses will be shown to scale approximately as the 4th power of the coil diameter.

We consider a simple case of two current carrying elements, parallel to the x axis at positions $y, z = (a, 0)$ and $(-a, 0)$ carrying currents $+NI$ amps and $-NI$ amps respectively.



the vector potential has only an x component as is parallel to the current elements. We calculate the vector potential at a point $(x, y, z) = (0, y, z)$ which is a distance a_1 from the current element at $y = a$, and a_2 from the element at $y = -a$

$$A_x(0,y,z) = \frac{\mu_0 NI}{4\pi}\int_{-\infty}^{\infty}\left(\frac{1}{r_1}-\frac{1}{r_2}\right)dx = \frac{\mu_0 NI}{2\pi}\int_0^{\infty}\left[\frac{1}{(a_1^2+x^2)^{1/2}}-\frac{1}{(a_2^2+x^2)^{1/2}}\right]dx$$

$$= \frac{\mu_0 NI}{2\pi}\left[\ln\left(\frac{r_1+x}{r_2+x}\right)\right]_{x=0}^{\infty} = \frac{\mu_0 NI}{2\pi}\ln\left(\frac{a_2}{a_1}\right)$$

the resultant magnetic fields are

$$B_x = 0$$

$$B_y = \frac{\partial A_x}{\partial z} = \frac{\mu_0 N I z}{2\pi}\left[\frac{1}{q_2^2} - \frac{1}{q_1^2}\right]$$

$$B_z = -\frac{\partial A_x}{\partial y} = -\frac{\mu_0 N I}{2\pi}\left[\frac{y+a}{q_2^2} - \frac{y-a}{q_1^2}\right]$$

the field in the center must be independent of coil size e.g 4.5T @ 4500A. Very approximately $a = 1.75''$ for the present coil ($= .044$m

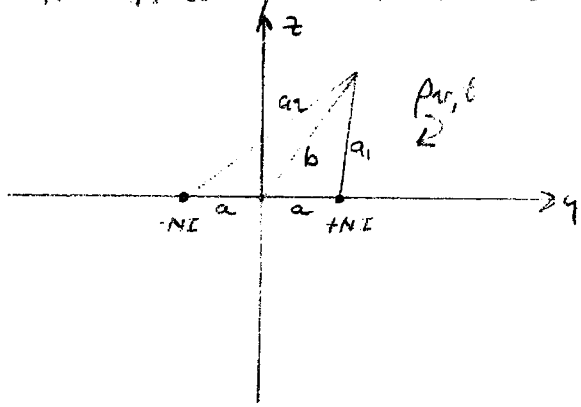
$$\therefore B_z(0,0,0) = -\frac{\mu_0 N I}{2\pi}\left[\frac{a}{q^2} + \frac{a}{q^2}\right] = \frac{\mu_0 N I}{\pi a}$$

$$\therefore N = \frac{\pi a B_z}{\mu_0 I} = \frac{10^{-3}\pi a}{\mu_0} \quad (= 110 \text{ turns for } a = .044\text{m})$$

$$\therefore B_y(y,z) = \frac{4.5\, I a z}{2 \times 4500}\left[\frac{1}{q_2^2} - \frac{1}{q_1^2}\right] = .0005\, I a z\left[\frac{1}{q_2^2} - \frac{1}{q_1^2}\right]$$

$$B_z(y,z) = .0005\, I a\left[\frac{y+a}{q_2^2} - \frac{y-a}{q_1^2}\right]$$

We now consider a conducting tube of thickness t, radius b, and volume conductivity with axis along x axis and $b > a$, and volume resistance ρ_{vr}



The eddy currents induced in the conducting tube during ramping ~~are~~ calculated from Faraday's Law

$$\vec{\nabla} \times \vec{E} = -\frac{d\vec{B}}{dt} = -\vec{\nabla} \times \frac{d\vec{A}}{dt} \Rightarrow \vec{E} = -\frac{d\vec{A}}{dt}$$

$$\therefore \; E_x = -\frac{\partial A_x}{\partial t} = -.0005\, a \ln\left(\frac{a_2}{a_1}\right) \frac{dI}{dt}$$

But $\vec{J} = \vec{E}/\rho_s = \frac{\vec{E}\, t}{\rho_v}$ where $\vec{J}$ = current per unit length of surface (amps/m) (current in element ds along circumference of tube)

$$\therefore \; J(y,z) = -\frac{.0005\, a t}{\rho_v} \cdot \ln\left(\frac{a_2}{a_1}\right) \frac{dI}{dt} \quad \text{amps/meter}$$

the heating in a strip of width ds along the surface of the conducting tube in the x direction is then

$$J^2(y,z)\, \frac{\rho_v}{t} \cdot ds = \left(\frac{.0005\, a t}{\rho_v}\right)^2 \ln^2\left(\frac{a_2}{a_1}\right)\left(\frac{dI}{dt}\right)^2 \frac{\rho_v}{t}\, ds \quad \text{watts/meter}$$

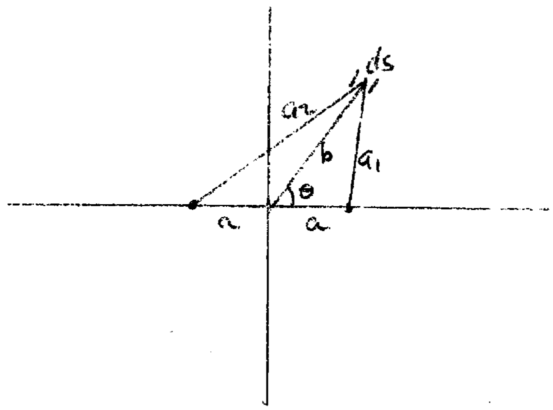
$$= \frac{2.5\times10^{-7} a^2 t}{\rho_v} \ln^2\left(\frac{a_2}{a_1}\right)\left(\frac{dI}{dt}\right)^2 ds \quad \text{watts/meter}$$

If we now consider a 2000 amp cycle at various dI/dt's then the total time is $\frac{2 \times 2000}{dI/dt}$ sec. Using a length of 6.4m for a magnet we get

$$dW = \frac{2.5\times10^{-7} a^2 t}{\rho_v} \times \ln^2\left(\frac{a_2}{a_1}\right) \times \left(\frac{dI}{dt}\right) \times 4000 \times 6.4\, ds \quad \text{Joules}$$

$$= 6.4\times10^{-3}\, \frac{a^2 t}{\rho_v} \ln^2\left(\frac{a_2}{a_1}\right) \frac{dI}{dt}\, ds \quad \text{joules.}$$

We now have to evaluate $\int_0^{2\pi b} \ln^2\left(\frac{a_2}{a_1}\right) ds = \int_0^{2\pi} \ln^2\left(\frac{a_2}{a_1}\right) b\, ds$



$$a_1^2 = a^2+b^2-2ab\cos\theta$$

$$a_2^2 = a^2+b^2+2ab\cos\theta$$

$$\ln\left(\frac{a_2}{a_1}\right) = \ln\left(\frac{1+\frac{2ab\cos\theta}{a^2+b^2}}{1-\frac{2ab\cos\theta}{a^2+b^2}}\right)^{1/2} = \frac{1}{2}\ln\left(\frac{1+u}{1-u}\right) \quad \text{where } u = \frac{2ab\cos\theta}{a^2+b^2}$$

but $\ln(1+u) = u - \frac{u^2}{2} + \frac{u^3}{3} - \frac{u^4}{4} \cdots$

$$\therefore \ln\left(\frac{a_2}{a_1}\right) = \frac{1}{2}\ln(1+u) - \frac{1}{2}\ln(1-u) = \frac{1}{2}\left[u - \frac{u^2}{2} + \frac{u^3}{3} + u - \frac{u^2}{2} + \frac{u^3}{3} - \cdots\right]$$

$$= \frac{1}{2}\left[2u + 2\frac{u^3}{3} + 2\frac{u^5}{5}\cdots\right]$$

$$= u + \frac{u^3}{3} + \frac{u^5}{5} \cdots$$

So $\int_0^{2\pi b} \ln^2\left(\frac{a_2}{a_1}\right) ds \sim \int_0^{2\pi} \left(\frac{2ab\cos\theta}{a^2+b^2}\right)^2 b\, d\theta = \frac{4\pi a^2 b^3}{(a^2+b^2)^2}$

For $b \gg a$ this is $\approx 4\pi a^2/b$

$\therefore$ in this limit $W = \frac{8.04\times10^{-2} a^4 t}{b\rho v} \frac{dI}{dt}$ joules/cycle

So approximately, cryostat heating is proportional to

a) $(\text{coil radius})^4$

b) $(\text{cryostat radius})^{-1}$

c) cryostat wall thickness

d) $(\text{cryostat material volume resistivity})^{-1}$

e) $\frac{dI}{dt}$

More exactly the energy per cycle is

$$W = \frac{8.04\times10^{-2}\, a^4 t}{b\rho_v}\,\frac{dI}{dt}\left[\frac{b^2}{4\pi a^2}\int_0^{2\pi}\ln^2\left(\frac{a_2}{a_1}\right)d\theta\right]$$

where the factor in brackets now depends only on the ratio b/a, and should approach 1 for $b/a \gg 1$

b/a	$\left[\frac{b^2}{4\pi a^2}\int_0^{2\pi}\ln^2\left(\frac{a_2}{a_1}\right)d\theta\right]$
[illegible]	[illegible]
[illegible]	[illegible]
1.6000	1.0180
1.8000	1.0110
2.0000	1.0071
2.2000	1.0048
2.4000	1.0034
2.6000	1.0025
2.8000	1.0019
3.0000	1.0014
3.2000	1.0011
3.4000	1.0009
3.6000	1.0007
3.8000	1.0005
4.0000	1.0004
4.2000	1.0004
4.4000	1.0003
[illegible]	[illegible]
[illegible]	[illegible]
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So the approximation is good to a few % down to $b/a \sim 1.2$

We now use the approximation to evaluate heating on a typical cryostat in an actual dipole

$$W = \frac{8.04\times10^{-2} a^4}{\rho_r} \frac{dI}{dt} \sum_i \frac{t_i}{b_i} \quad \text{Joules}$$

where a = coil radius = 1.75" = .044m

ρ_r = $.53\times10^{-6}$ ohm-m for stainless

$\frac{dI}{dt}$ = 400 amps/sec

	b_i	t_i	t_i/b_i	
1φ tube	2.85"	.049"	17.2×10^{-3}	
2φ tube	3.00"	.036"	12×10^{-3}	
Inner shield tube	3.24"	.075"	23.1×10^{-3}	*
Outer shield tube	3.38"	.036"	10.7×10^{-3}	*
Vacuum tube	3.74"	.036"	9.6×10^{-3}	*
			72.6×10^{-3}	

$$\therefore W = \frac{8.04\times10^{-2}\times(.044)^4\times400\times72.6\times10^{-3}}{.53\times10^{-6}} = 16.5 \text{ joules/cycle}$$

this number is to be compared to 26 Joules per cycle predicted from inductance measurements for assembled magnets with iron yokes (see report # UPC 31). Roughly 80% or 21 Joules/cycle was expected to be due to the cryostat, and the remainder to the bore tube (12%) and the collars (8%).

* not part of He system

We now consider the bore tube heating. In this case

$B_z = -10^{-3} I$ e.g. 4500 amps yields 4.5 T uniform field

$\therefore A_x = +\ 10^{-3} I y$

$\therefore E_x = -\dfrac{\partial A_x}{\partial t} = 10^{-3} y \dfrac{dI}{dt}$

$J_x = \dfrac{E_x t}{\rho_r} = \dfrac{10^{-3} y t}{\rho_r} \dfrac{dI}{dt}$ amps/m

$\therefore$ heating in a strip of width ds on a bore tube of radius b and thickness t is:

$$dW = J^2 \frac{\rho_r}{t} ds = \frac{10^{-6} y^2 t^2}{\rho_r^2} \frac{\rho_r}{t} \left(\frac{dI}{dt}\right)^2 ds \quad \text{watts/meter}$$

we again assume a 2000 amp cycle for a 6.4m magnet

$$W = \int_0^{2\pi} \frac{10^{-6} y^2 t}{\rho_r} \left(\frac{dI}{dt}\right)^2 \frac{2 \times 2000}{(dI/dt)} \times 6.4\ b\, d\theta$$

using $y = b\cos\theta$ and $\int_0^{2\pi} \cos^2\theta\, d\theta = \pi$

$$W = 10^{-6} \times 4000 \times 6.4 \frac{b^3 t}{\rho_r} \frac{dI}{dt} \pi \quad \text{Joules per cycle}$$

$$= 8.04 \times 10^{-2} \frac{b^3 t}{\rho_r} \frac{dI}{dt} \quad \text{Joules/cycle.}$$

using $b = 1.3''(.033\text{m})$; $t = .061''(1.55 \times 10^{-3}\text{m})$; $\rho_r = .53 \times 10^{-6}$ ohm-m and $dI/dt = 400$ A/sec

$W = 3.38$ Joules/cycle (Scales as (tube radius)^3 but independent of coil dimensions

(roughly 20% of cryostat heating).

We now summarize the eddy current heating for $I_{max} = 2000$ amp cycle assuming $B = 10^{-3} I$ 4.5T for 4500A and 6.4m long dipoles

$$W = 8.04 \times 10^{-2} \frac{dI}{dt} \Bigg[\underbrace{\sum_i \frac{a^4 t_i}{\rho_{ri} b_i}}_{\text{outside the coils}} + \underbrace{\sum_i \frac{b_i^3 t_i}{\rho_{ri}}}_{\text{inside the coils}} \Bigg] \text{ Joules/cycle}$$

a : coil radius (m)

b = tube radii (m)

t = tube thickness (m)

ρ_r : resistivity of tube material (ohm-m)

this algorithm of course does not use the exact coil geometry and does not include the iron yoke directly (although we compensated for it in our value for N on page 2) so should be considered only approximate.