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Sensitivity of Beam Position Detectors for the Tevatron

The purpose of this note is to calculate the sensitivity of various geometry wall-current or electrostatic beam position detectors for possible use in the Tevatron. We develop the potential for an off center line charge in a grounded conducting circular tube and calculate the azimuthal current density. From this we can calculate the induced currents on various size electrodes.

The harmonic expansion for a line charge potential for a charge q per unit length at a position r_0, θ_0 in a cylindrical coordinate system is for $r > r_0$ (see Smythe section 4.02):

$$V_0(r,\theta) = \frac{q}{2\pi\epsilon_0}\left[\sum_{n=1}^{\infty} \frac{1}{n}\left(\frac{r_0}{r}\right)^n(\cos n\theta_0 \cos n\theta + \sin n\theta_0 \sin n\theta) - \ln r\right]$$

Similarly, the potential at r,θ from another line charge (image charge) $-q$ per unit length situated at r_1, θ_0, where $r_1 > r$, is:

$$V_1(r,\theta) = \frac{-q}{2\pi\epsilon_0}\left[\sum_{n=1}^{\infty} \frac{1}{n}\left(\frac{r}{r_1}\right)^n(\cos n\theta_0 \cos n\theta + \sin n\theta_0 \sin n\theta) - \ln r_1\right]$$

Each of these potentials satisfies Laplace's equation, as does a linear sum plus a constant:

$$V_s(r,\theta) = \frac{q}{2\pi\epsilon_0}\left[\sum_{n=1}^{\infty} \frac{1}{n}\left[\left(\frac{r_0}{r}\right)^n - \left(\frac{r}{r_1}\right)^n\right](\cos n\theta_0 \cos n\theta + \sin n\theta_0 \sin n\theta) - \ln(r/r_1) + \text{const}\right]$$

We can make this potential vanish at $r=a$ if $\frac{r_0}{a} = \frac{a}{r_1}$ and $\text{const} = \ln(a/r_1)$

$$\therefore V_s(r,\theta) = \frac{q}{2\pi\epsilon_0}\left[\sum_{n=1}^{\infty} \frac{1}{n}\left[\left(\frac{r_0}{r}\right)^n - \left(\frac{r_0 r}{a^2}\right)^n\right](\cos n\theta_0 \cos n\theta + \sin n\theta_0 \sin n\theta) - \ln(r/a)\right]$$

represents the potential of a line charge or beam in a grounded conducting tube of radius a.

the surface charge density on the inside surface of this conducting tube is then

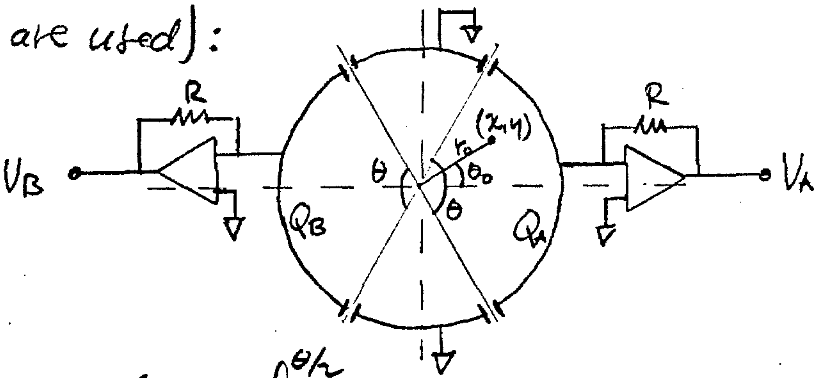
$$\sigma(\theta) = -\hat{r}\cdot\epsilon_0\vec{E}\Big|_{r=a} = -\hat{r}\cdot(-\epsilon_0\vec{\nabla}V)\Big|_{r=a} = +\epsilon_0\,\frac{\partial V(r,\theta)}{\partial r}\Big|_{r=a}$$

$$= \frac{-q}{2\pi a}\left[1 + 2\sum_{n=1}^{\infty}\left(\frac{r_0}{a}\right)^n(\cos n\theta_0\cos n\theta + \sin n\theta_0\sin n\theta)\right]$$

If we now consider the total charge Q_A between $-\theta/2$ and $+\theta/2$, and the total charge Q_B between $\pi-\theta/2$ and $\pi+\theta/2$ we have (we assume that all surfaces remain at ground potential - this assumes that if these charges are to be measured, that charge sensitive amplifiers are used):

$$V_A = R\,\frac{dQ_A}{dt}$$

$$V_B = R\,\frac{dQ_B}{dt}$$



$$Q_A = \int_{-\theta/2}^{\theta/2} a\,\sigma(\theta)\,d\theta = -\frac{q\theta}{2\pi}\left[1 + \frac{2}{\theta}\sum_{n=1}^{\infty}\left(\frac{r_0}{a}\right)^n\int_{-\theta/2}^{\theta/2}(\cos n\theta_0\cos n\theta + \sin n\theta_0\sin n\theta)\,d\theta\right]$$

$$= -\frac{q\theta}{2\pi}\left[1 + \frac{4}{\theta}\sum_{n=1}^{\infty}\frac{1}{n}\left(\frac{r_0}{a}\right)^n\cos n\theta_0\sin\frac{n\theta}{2}\right]$$

and

$$Q_B = -\frac{q\theta}{2\pi}\left[1 + \frac{4}{\theta}\sum_{n=1}^{\infty}\frac{1}{n}\left(\frac{r_0}{a}\right)^n\cos n\theta_0\,\sin\left(\pi + \frac{n\theta}{2}\right)\right]$$

We are now in a position to investigate displacement sensitivities and non-linearities for various geometry position sensitive beam monitors.

We select the pickup electrode geometry by setting a and θ, and the beam position by setting r_0 and θ_0 (or $x = r_0 \cos\theta_0$ and $y = r_0 \sin\theta_0$). We measure the sensitivity by calculating

$$S = \frac{(Q_A - Q_B)}{(Q_A + Q_B)}$$

at each point.

We first check the validity of this calculation by calculating the parameter S for the KEK wall-current detector as described in RSI 50, pg 450 (1979), using $\theta = 90°$ and $a = 68$ mm. these results are to be compared to figure 2 of that paper. the agreement is quite good.

$S(x,y)$ for $a = 68$ mm, $\theta = 90°$

y (mm)	X (mm) 5mm	10mm	15mm	20mm	25mm	30mm	35mm	40mm	45mm	50mm
50.00	0.127	0.252	0.373	0.489	0.597	0.698	0.790	0.873	0.875	
45.00	0.131	0.260	0.385	0.503	0.613	0.715	0.807	0.889	0.958	0.990
40.00	0.134	0.265	0.391	0.510	0.620	0.720	0.808	0.883	0.941	0.979
35.00	0.135	0.267	0.394	0.512	0.621	0.717	0.801	0.870	0.924	0.961
30.00	0.135	0.268	0.394	0.511	0.617	0.710	0.790	0.856	0.907	0.946
25.00	0.135	0.267	0.391	0.507	0.610	0.701	0.779	0.842	0.893	0.933
20.00	0.134	0.265	0.388	0.502	0.604	0.693	0.768	0.831	0.881	0.921
15.00	0.133	0.263	0.385	0.497	0.598	0.685	0.759	0.821	0.872	0.913
10.00	0.132	0.261	0.382	0.493	0.593	0.679	0.753	0.814	0.865	0.907
5.00	0.132	0.260	0.381	0.491	0.590	0.675	0.749	0.810	0.861	0.903
0.00	0.132	0.260	0.380	0.490	0.589	0.674	0.747	0.809	0.860	0.901

It should be noted that there is a slight increase in sensitivity to x displacements as y is increased i.e. moved away from the median plane.

We now examine the sensitivity of a possible Tevatron pickup geometry, $\theta = 180°$ and $a = 33$ mm (to match the beam pipe average radius) {this agrees fairly well with measurements in the median plane of a device designed by Ed Higgins}:

$S(x,y)$ for $a = 33$ mm, $\theta = 180°$

y(inches) \ X(inches)	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0"
1.00	0.233	0.430	0.583	0.701	0.793	0.868	0.930	0.992		
0.90	0.185	0.353	0.497	0.615	0.713	0.796	0.866	0.927	0.981	
0.80	0.156	0.303	0.435	0.550	0.649	0.735	0.811	0.877	0.936	0.983
0.70	0.137	0.269	0.390	0.500	0.599	0.686	0.764	0.833	0.895	0.952
0.60	0.124	0.244	0.358	0.463	0.559	0.646	0.725	0.797	0.861	0.920
0.50	0.115	0.227	0.334	0.435	0.528	0.615	0.694	0.767	0.833	0.894
0.40	0.108	0.214	0.316	0.414	0.505	0.591	0.670	0.743	0.810	0.872
0.30	0.103	0.205	0.304	0.398	0.488	0.573	0.652	0.725	0.793	0.856
0.20	0.100	0.199	0.295	0.388	0.477	0.560	0.639	0.713	0.781	0.844
0.10	0.098	0.196	0.291	0.382	0.470	0.553	0.632	0.705	0.774	0.838
-0.00	0.098	0.194	0.289	0.380	0.468	0.551	0.629	0.703	0.771	0.835

We also calculate the sensitivity of a $\theta = 90°$ $a = 33$ mm pickup geometry:

$S(x,y)$ for $a = 33$ mm, $\theta = 90°$

y(inches) \ X(inches)	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
1.00	0.131	0.259	0.383	0.501	0.611	0.713	0.802	0.892		
0.90	0.136	0.269	0.397	0.519	0.632	0.736	0.830	0.914	0.991	
0.80	0.139	0.276	0.406	0.529	0.643	0.746	0.836	0.912	0.967	
0.70	0.141	0.279	0.411	0.533	0.645	0.744	0.829	0.897	0.947	0.979
0.60	0.142	0.280	0.411	0.532	0.641	0.737	0.817	0.881	0.929	0.964
0.50	0.141	0.279	0.409	0.528	0.635	0.727	0.804	0.866	0.914	0.950
0.40	0.141	0.277	0.405	0.523	0.627	0.717	0.792	0.853	0.901	0.939
0.30	0.140	0.275	0.402	0.518	0.620	0.708	0.782	0.842	0.891	0.930
0.20	0.139	0.273	0.399	0.513	0.614	0.701	0.775	0.835	0.884	0.924
0.10	0.138	0.272	0.397	0.510	0.611	0.697	0.770	0.830	0.880	0.920
-0.00	0.138	0.271	0.396	0.510	0.610	0.696	0.769	0.829	0.878	0.918

the data for these two geometries are plotted in Figs 1 and 2. Note that the $\theta = 90°$ geometry is somewhat more sensitive than the $\theta = 180°$ geometry, but at the expense of signal size. In the 90° geometry, only about 1/2 the charge is seen. the 180° geometry however suffers from more nonlinearity along y.

In Figure 3 we plot $\frac{S(x,y)}{X}$ vs x at $y=0$ for various θ. It is apparent that the most linearity along X is obtained with $\theta = 180°$. In Figure 4 we plot $\frac{S(x,y)}{X}$ vs y at $x = 0.1''$ for various θ. In this case optimum linearity is obtained when $\theta \sim 90°$. An optimum θ for linearity over a finite area in (x,y) is probably $\sim 120°$ to $150°$.

In summary, the three parameters

a) deflection sensitivity near the center
b) linearity in the required area of (x,y)
c) maximum signal size

cannot be simultaneously optimized. We therefore need to make some decisions as to how much to compromize on which parameters. this is best done before building hardware.

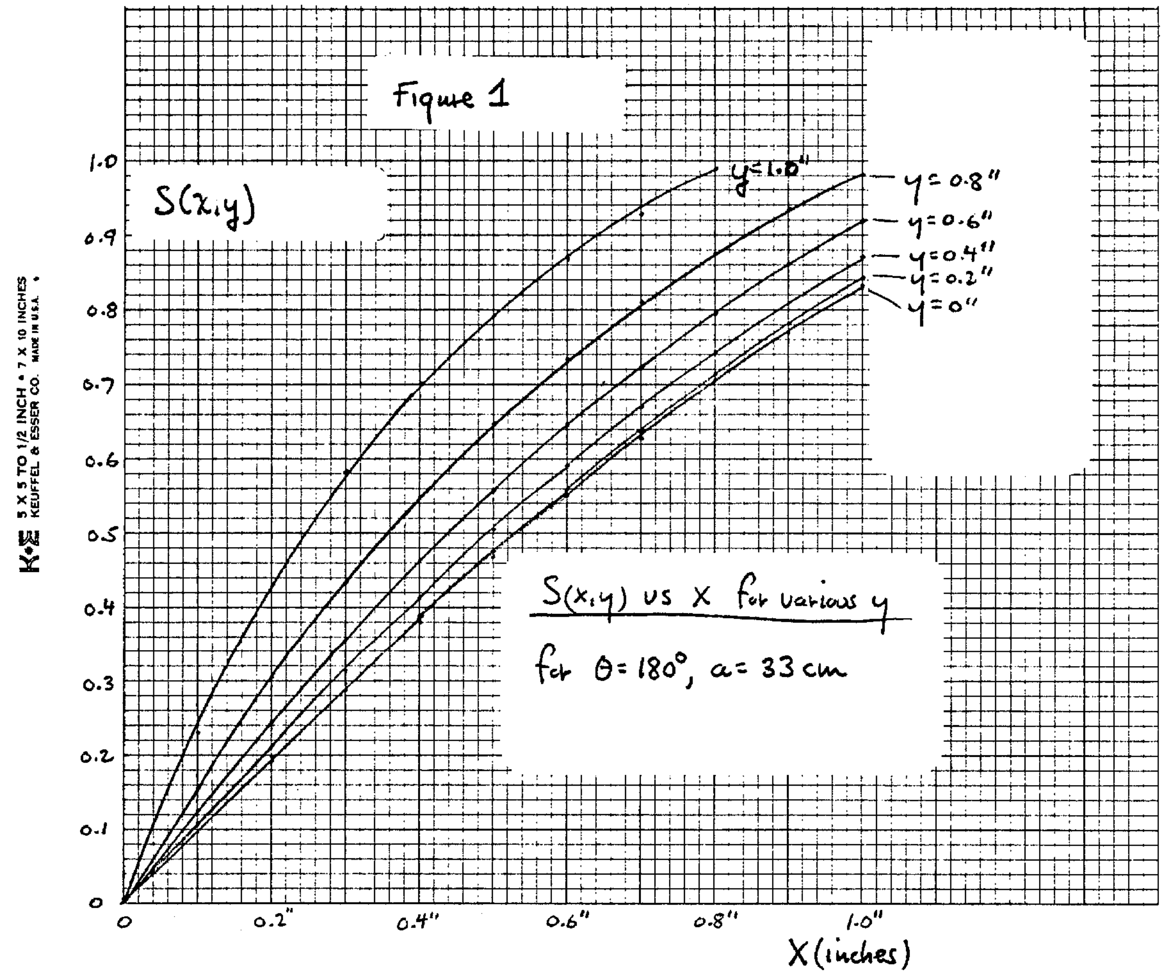


Figure 1
S(x,y)
y=1.0"
y=0.8"
y=0.6"
y=0.4"
y=0.2"
y=0"
S(x,y) vs X for various y
for θ=180°, a=33 cm
X (inches)
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1.0
0.2"
0.4"
0.6"
0.8"
1.0"
5 X 5 TO 1/2 INCH • 7 X 10 INCHES
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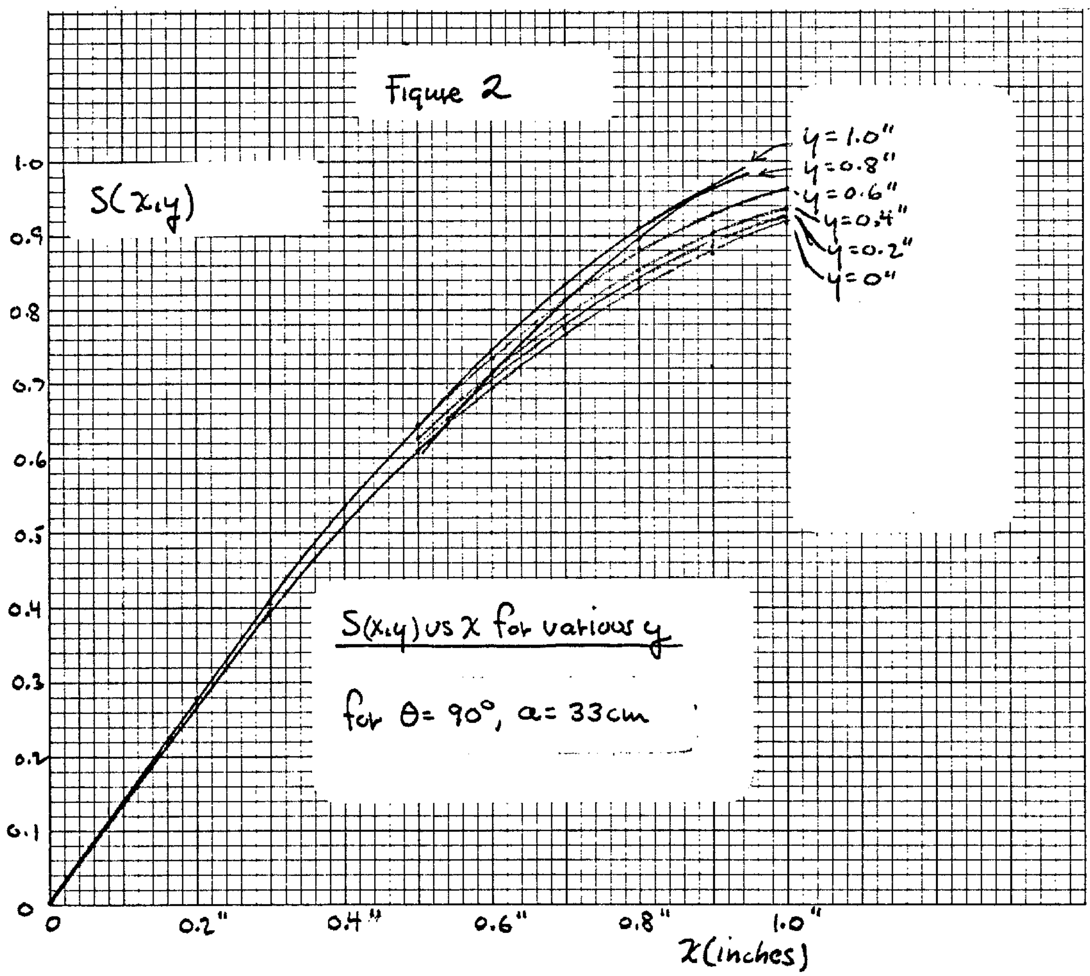
Figure 2
S(x,y)
y=1.0"
y=0.8"
y=0.6"
y=0.4"
y=0.2"
y=0"
S(x,y) vs x for various y
for θ=90°, a=33cm
x (inches)
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1.0
0.2"
0.4"
0.6"
0.8"
1.0"

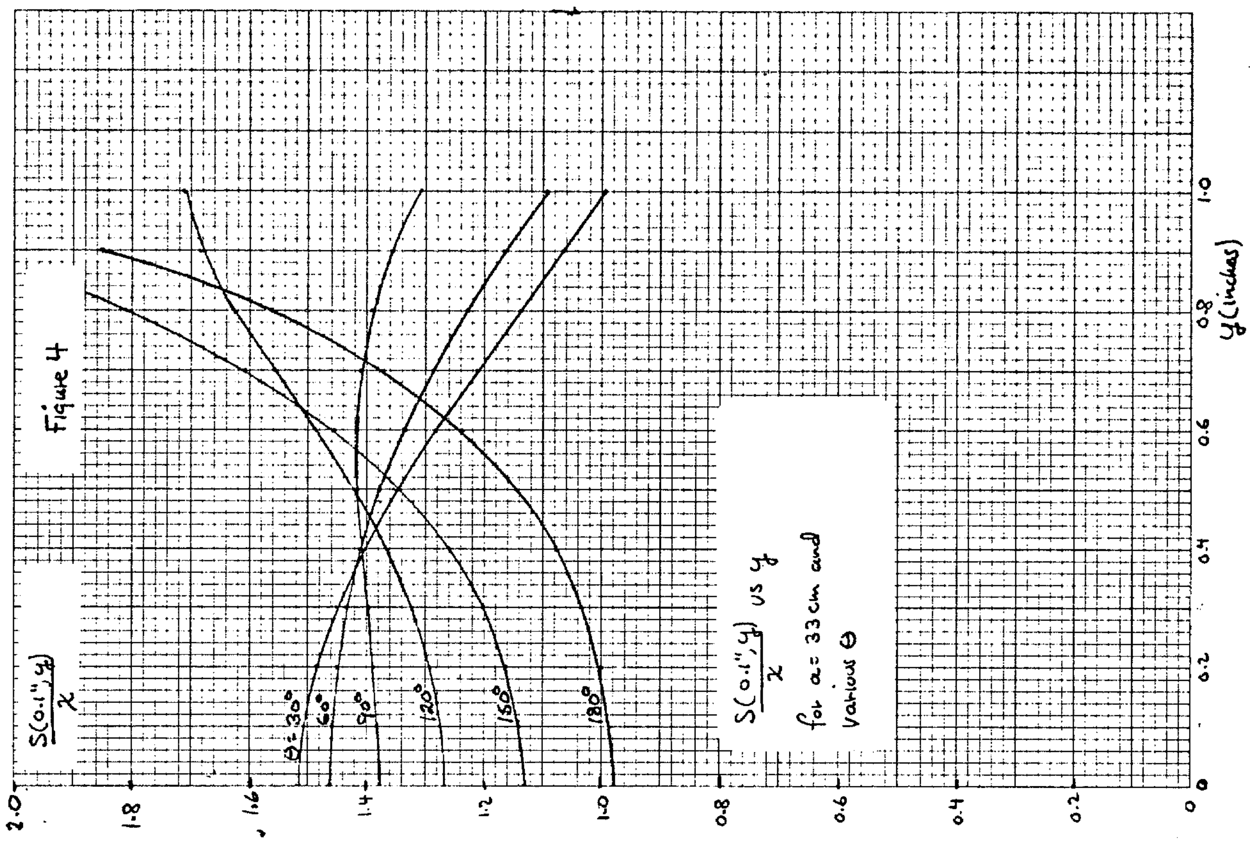
Figure 4
$\frac{S(0.1'',y)}{x}$
θ = 30°
60°
90°
120°
150°
180°
$\frac{S(0.1'',y)}{x}$ vs y
for a = 33 cm and
various θ
y (inches)
0
0.2
0.4
0.6
0.8
1.0
1.2
1.4
1.6
1.8
2.0

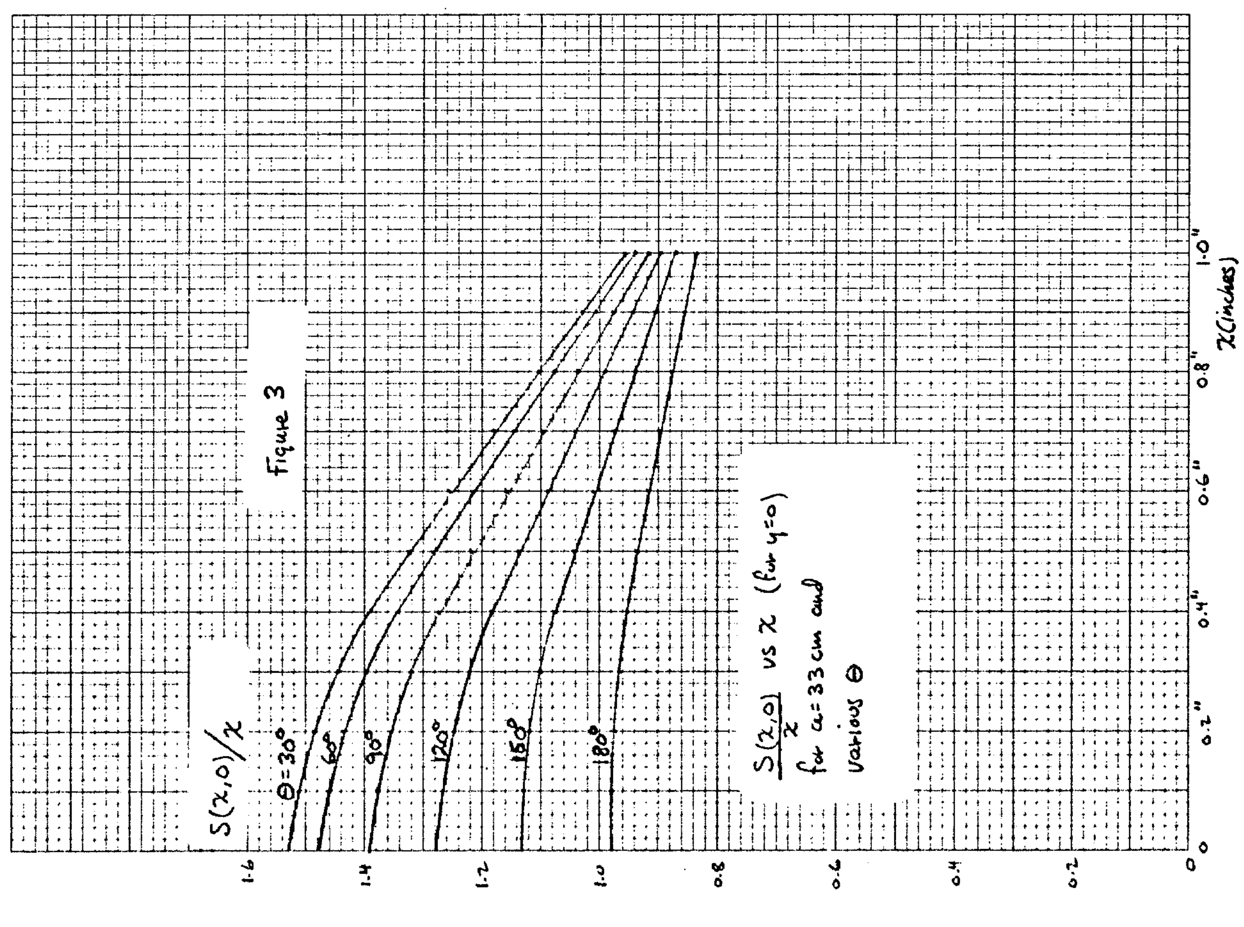
Figure 3
S(x,0)/x
θ = 30°
60°
90°
120°
150°
180°
$\frac{S(x,0)}{x}$ vs x (for y=0)
for a = 33 cm and
various θ
x (inches)
0
0.2"
0.4"
0.6"
0.8"
1.0"
0.2
0.4
0.6
0.8
1.0
1.2
1.4
1.6