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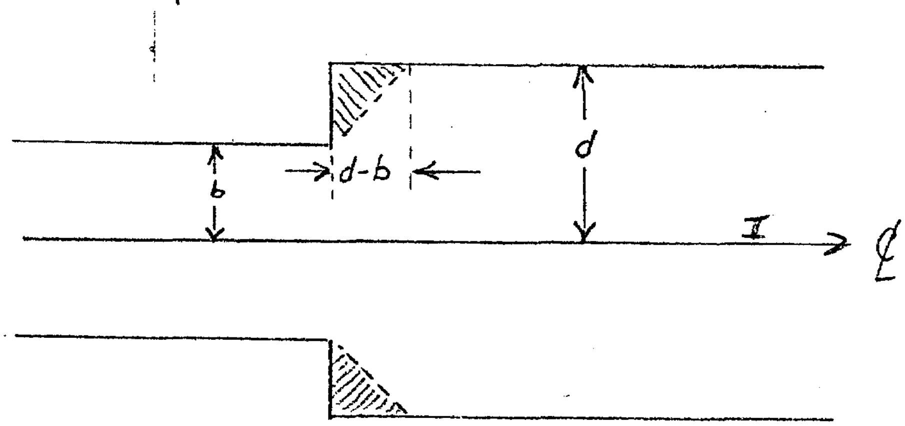
On Reducing the Coupling Impedance of Step Discontinuities in Vacuum Chambers

We consider the possibility that by filling in the corner of a step discontinuity in a vacuum chamber with conducting material, the coupling impedance of the step discontinuity can be substantially reduced.

Consider a step of the form



where b and d = the radii of the circular vacuum chamber on both sides of the discontinuity. We assume that the magnetic fields generated by the beam are allowed to penetrate the 45° shaded area at the discontinuity, but that the electric fields are prevented from entering the shaded area by the electrostatic shielding of the adjacent walls. We wish to calculate the effective impedance of the shaded area.

the inductance of the shaded area is given by

$$L = \frac{1}{I^2} \iiint \frac{B^2}{\mu_0} \, r \, dr \, d\theta \, dz \qquad [1]$$

where $B = \dfrac{\mu_0 I}{2\pi r}$

then
$$L = \frac{\mu_0}{(2\pi)^2} \int_0^{d-b} \int_{b+z}^{d} \int_0^{2\pi} d\theta \; \frac{dr}{r} \; dz \qquad [2]$$

$$= \frac{\mu_0}{2\pi} \int_0^{d-b} \ln(d) - \ln(b+z) \; dz$$

$$= \frac{\mu_0}{2\pi} \left[(d-b)\ln(d) - d\ln(d) + d + b\ln(b) - b \right]$$

$$L = \frac{\mu_0}{2\pi} \left[(d-b) - b\ln(d/b) \right] \qquad [3]$$

We tabulate the value of $\frac{L}{\mu_0} \frac{b}{(d-b)^2}$ for 3 models:

the 45° model (above); the Keil-Zotter[1] model; and the Markcuvitz[2] model:

$$L_{KZ} = .241 \mu_0 \frac{(d-b)^2}{d + .412b} \qquad [4]$$

$$L_M = \frac{\mu_0}{(2\pi)^2} \frac{(d-b)^2}{b} \left[1 - 2\ln\left(\frac{d-b}{2b}\right) \right] \qquad [5]$$

Step size $\frac{d-b}{b}$	$L \times \frac{b}{\mu_0 (d-b)^2}$ 45°	Keil-Zotter	Markcuvitz
.1	.074	.159	.177
.2	.070	.150	.142
.5	.060	.126	.096

Which leads us to ask whether the coupling impedance of a step discontinuity can be reduced substantially by "filling-in" part of the step with conducting material - e.g. a taper.

Specifically, in the case of the bellows, $b \sim 0.035$m, $d \sim .042$ m leading to an estimate of $\frac{d-b}{b} = 0.2$ for the step size, and an inductance of (using the Markuvitz values)

$$L = 0.142 \mu_0 \frac{(d-b)^2}{b} = 2.5 \times 10^{-10} \text{ Hy per step}$$

assuming roughly 1000 bellows (2000 steps) and $\omega_0 = 3 \times 10^5$/sec, the estimated contribution to $(Z/n)_{\parallel}$ is

$$(Z/n)_{\parallel} = i\, 2000\, \omega_0 L = i\, 2000 \times 3 \times 10^5 \times 2.5 \times 10^{-10}\, \Omega$$
$$= i\, 0.15 \text{ ohms.}$$

As the bellows have machined end flanges, they are now being machined with 45° tapers. It would be useful in the future however if someone were to calculate more quantitatively what tapered discontinuities could really do.*

1. Keil and Zotter; Particle Accelerators 3 page 11 (1972) eqn 2.13
2. Hahn and Zatz; IEEE NS # 26 #3 page 3626 (1979)

* We have roughly 250 position detector assemblies where the inductance per step is $\approx 10^{-9}$ Hy. $\therefore (Z/n)_{\parallel} \sim i\, 500 \times 3 \times 10^5 \times 10^{-9} = i\, .15\, \Omega$. Should we taper the ends?