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COMMENTS ON THE BEAM-BEAM INTERACTION IN STORAGE RINGS*

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I. Introduction

Beam-beam interaction, which is generally considered to be the most important factor in determining the lifetime of colliding beams, is nonlinear in its essential characteristics. There is no "solution" for the beam dynamics when beam-beam interaction is involved, at least no solution in the sense that there is a solution for the beam dynamics in ordinary fixed-target accelerators. There exist of course nonlinear fields in any real accelerators even without beam-beam interaction. The crucial difference is that, in fixed-target accelerators, the duration of beam is a matter of seconds while it is several hours or more in storage rings. The most important approximation in the standard treatment of nonlinear beam dynamics is to consider each resonance separately. The Hamiltonian is then integrable and the solution can be found to give the stable phase space area and the resonance width. The approximation is quite adequate for practically any type of beam behaviors in ordinary accelerators, a typical example being resonance extraction. In contrast with this, it has been clearly established by now that the single-resonance approximation cannot explain the observed beam lifetime in storage rings both of electrons and protons.[1] One must deal with non-integrable Hamiltonian systems and there is as yet no established general formalism that can lead to solutions. I am not happy that it is necessary to state this simple fact before anything else; some people (influential ones at that) at Fermilab seem to suspect or even believe that solutions are known in

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* This report is meant to be a supplement to TM-910, "Field Quality of Doubler Dipoles and Its Possible Implications", October 15, 1979. Nonspecialists may benefit from reading Section V of the report.

the East and in the West but not in Illinois.

The purpose of this note is to explain the recent work by S. Kheifets at SLAC[2] and also a similar work by Sandro Ruggiero which is still in its early stage of development.* The work by Kheifets has been cited by some people (much to the dismay of Sam Kheifets himself, I am sure) to scare innocent folks at Fermilab as a conclusive proof that the colliding project at Fermilab is doomed to be a failure from the start. The work is certainly very interesting but, for all I know, it simply is not possible to predict in a convincing manner the beam lifetime of proton storage rings with his formula. The fact that I cannot predict the beam lifetime is perhaps not important (except, of course, for me) but it is important to recognize the truth that nobody can unless one takes shots in the dark seriously. For us, there are only two choices available: either forget about the doubler as a colliding device or do whatever we can and follow the advice given by a fair lady of Venice long time ago, "You must take your chance;".

During the past ten months or so, I have discussed the problem many times with Sandro Ruggiero. Although we disagree in some technical details, I believe we agree in the essential points. I am gratefult to him for his unceasing effort to enlighten me on this subject.

II. Before Kheifets-Ruggiero Models

It is worthwhile to review briefly the situation before Kheifets and then Ruggiero proposed new models of beam loss mechanism. Since the single-resonance model cannot explain the observed beam lifetime, some other mechanism must be considered and, broadly speaking, there have been two entirely different lines of investigation.

The first is the model proposed by M. Month[3] in which one still retains the single-resonance concept but the relevant parameters modulate in time with a certain frequency. One may call it a dynamic single-

* The fact that the work has not yet been finished makes me hesitate to express any opinion on it. Here I am taking advantage of the permission given to me graciously by Sandro. I am of course totally responsible for any errors contained in my description of his work. I understand a significant contribution to the basic idea of this work has been made by Fred Mills.

resonance model but some people refer to it as the trapping model. One obvious candidate of the time-varying parameters is the tune of betatron oscillation. The familiar phase-space picture (see Fig. 1) with its central stable area and the surrounding outer stable islands (N islands for the N-th order resonance) now expands and contracts more or less periodically.

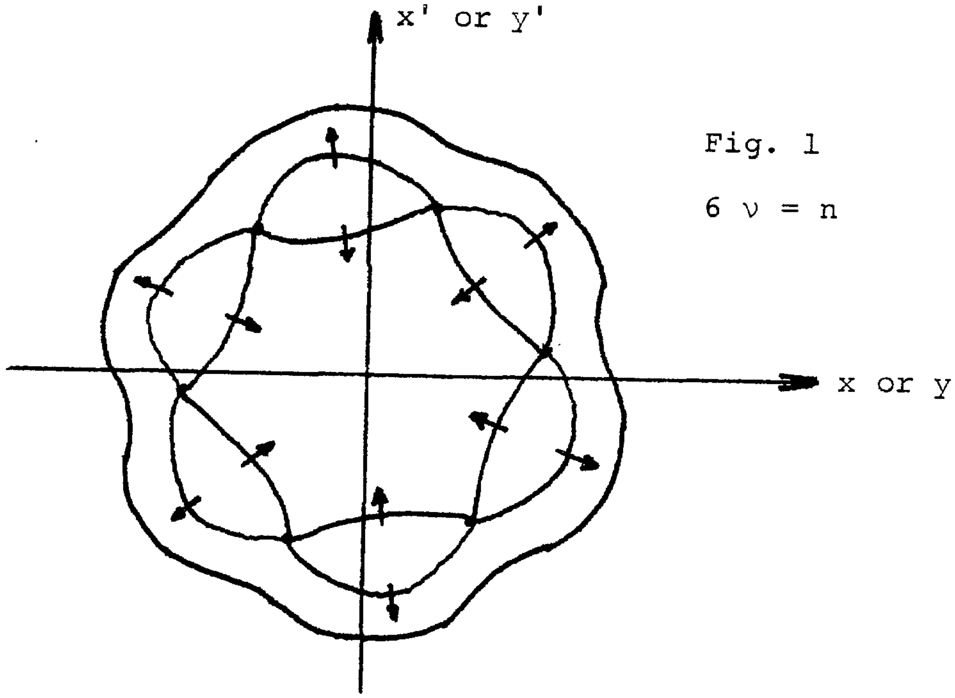

Fig. 1

$6\nu = n$

A particle may have a certain probability of getting trapped by these moving islands and then deposited at places in the phase space far away from the central stable area. The quantitative description of this process must include not only the total area of islands but the instantaneous growth rate of the amplitudes of islands also. For example, if the motion of an island is so slow that particles go around the phase space many times before the position of the island changes significantly, the process is adiabatic and particles will just follow the islands. There will be no depositing of particles. In the opposite situation, islands will move in and out without trapping particles. The model proposes that the probability of trapping is proportional to the area of islands and decreases exponentially with the growth rate of the amplitude. Because of the characteristics of resonances, particles are trapped when the amplitude is small and dropped from islands when they become leaky at large amplitudes. There will be a continuous pumping-out of particles from inner to outer area of the central stable region. It is of course difficult to estimate quantitatively the trapping probability for a given situation. One probably uses an expression with unknown parameters and determine these parameters either from experimental data or from numerical simulations. The idea is certainly ingenious, perhaps too much so for the taste of some people. In this model, crucial parameters for the beam lifetime are power supply ripples, synchrotron oscillation frequency, chromaticity and the magnetic aperture of dipoles and quadrupoles.

Another type of dynamic single-resonance model tries to incorporate the diffusion-like behavior of beam parameters into the single-resonance picture. In a way, this model may be regarded as a precursor of the Kheifets-Ruggiero models except that one still tries to salvage the concept of single resonance. I remember I read the report by Hereward[4] some years ago but nothing remains in my mind. Consequently, I cannot talk about this model with any degree of comfortableness. Alex Chao states in his report[1]: "Some attempts have been made in this direction but unfortunately hindered by the very difficult mathematics." In view of the renewed interest in diffusion-like behaviors of the beam, I should probably look at Hereward's report again and compare with Kheifets-Ruggiero models. Let me simply quote two paragraphs from the Introduction of Hereward's paper. I find the underlined sentence (underline is mine) to be particularly interesting since it clearly reveals the fundamental difference between Hereward's view and Ruggiero's view of the same phenomenon.

" Consider the combined effect of nonlinear betatron resonances and scattering processes such as gas scattering, intra-beam scattering and quantum emission in electron machines. The diffusion of particles across the separatrices and deformed trajectories may produce effects which would not occur if one had the resonances or the scattering alone."

" The more refined theory of nonlinear motion shows that Arnol'd diffusion and stochastic layers can occur in the absence of any indeterminism in the equations of motion. <u>But it seems reasonable to suppose that a sufficient amount of scattering will "cover up" those phenomena, and put one in a region where the classical resonance theory is adequate but needs to be combined with the effects of the scattering</u>."

As soon as one abandons the concept of the single resonance as something inadequate, one cannot avoid facing the true difficulty of nonlinear motions. Since there is no mathematical solution, one must either resort to numerical simulations or try to find some analytic guidance with quantities that can be calculated from the single-resonance model. A great deal of work has been done numerically by many people and even a brief review is impossible for me. There are many technical difficulties involving not only the question of accuracies but also the question of what parameters are relevant. The running time is usually very long even for one to ten million revolutions. For the doubler, ten million revolutions correspond to "the first three minutes". According to Keil at CERN, the latest round of simulation studies will be done with the CRAY-1 computer at Daresbury which is known to be 150 times faster than CDC6600 for this type of problem.

I am afraid it is beyond my ability to make useful comments on the mathematical progresses made in this field. What follows is simply a sketch of the standard procedures accelerator physicists have been following, sometimes blindly. A good, readable review article has been written by B. Chirikov, "A Universal Instability of Many-Dimensional Oscillation Systems" (to be published in Physics Reports). In order to explain the meaning of "stochastic layers" in phase space, let me take a nonlinear mapping, which is often called "the standard mapping". Coordinates are (r,θ) and the transformation from (r_n,θ_n) to (r_{n+1},θ_{n+1}) is

$$r_{n+1} = r_n - (k/2\pi)\sin(2\pi\theta_n),$$

$$\theta_{n+1} = \theta_n + r_{n+1} .$$

With suitable definition of variables, accelerator physicists will say this is simply a problem of stationary rf localized in a ring. The parameter k is the strength of nonlinearity; if $k=0$, the problem is clearly integrable (r = const.). For $k \neq 0$, there are three types of orbits as shown in Fig. 2 (see next page). Two dots represent periodic orbits which can be regarded as orbits with rational tune values. Two continuous lines labelled A and B are examples of the celebrated KAM surfaces. The theorem of Kolmogorov, Arnol'd and Moser assures us that, for suffi-

J. M. Greene, PPPL-1489 (Princeton Plasma Laboratory), Nov. '79

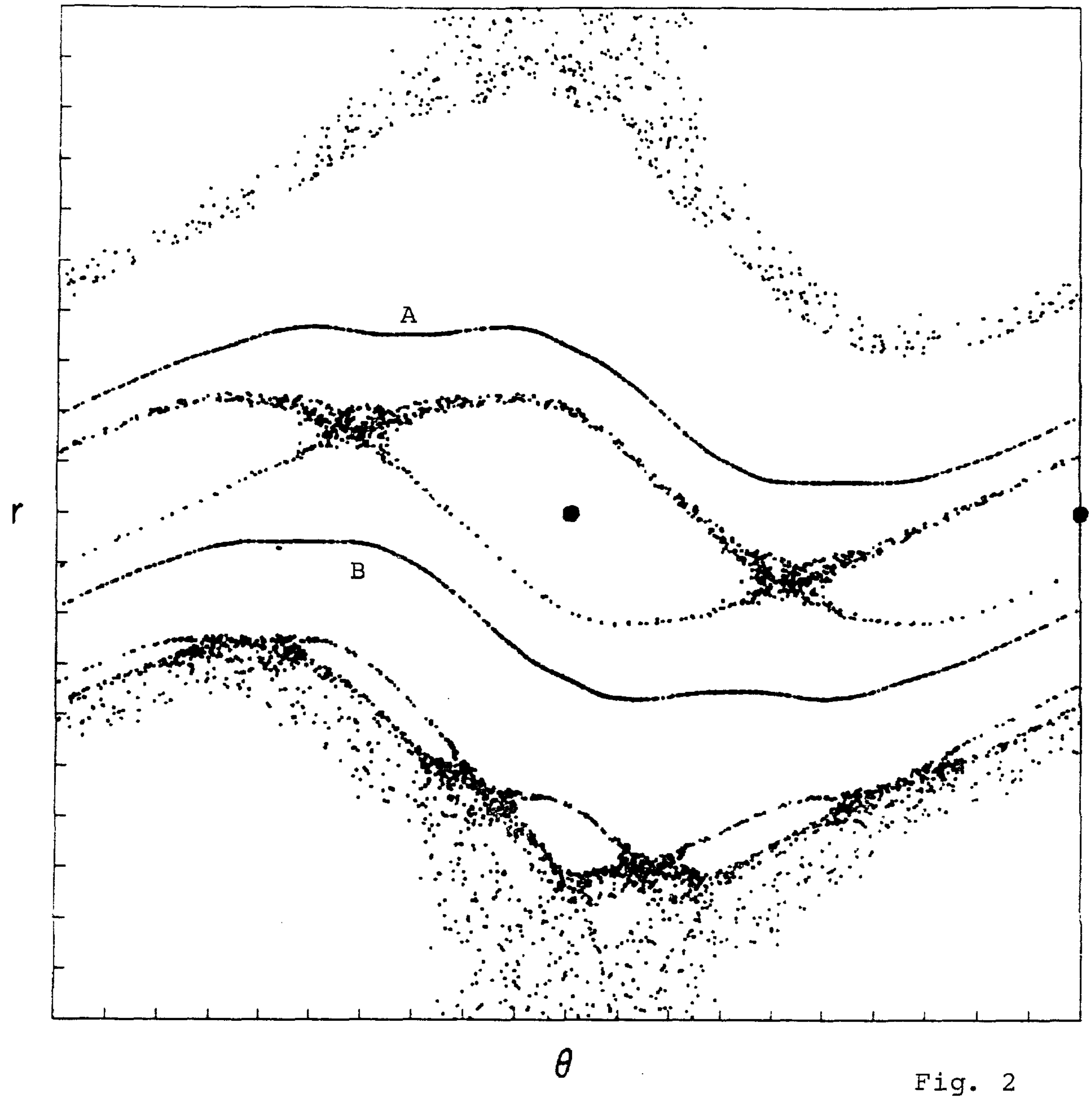

Fig. 2

ciently small but nonzero values of k, this type of orbits exist and, in the limit of $k \to 0$, they go continuously into orbits with irrational tune values. The third type of orbits, which did not exist when $k=0$ (integrable case), appear for $k \neq 0$ and they seem to fill some portions of phase space randomly. They are the ones called the stochastic orbits or the stochastic layers. One of the fundamental problems in nonlinear dynamics is to find, for a given system, the limiting value of k above which no KAM surfaces exist and the entire phase space becomes stochastic. It should be emphasized that KAM surfaces are "impenetrable". In spite of stochastic layers in-between, particles can never cross KAM surfaces. Because of this, one says the beam should be stable; the randomness in phase is immaterial as far as the beam stability is concerned. It is not easy to find the limiting strength of nonlinearity even with numerical studies. J. Greene investigated the standard mapping and obtained the limiting value $k = 0.9716$ using the hypothesis that the disapperance of a KAM surface is associated with a sudden change from stability to instability of nearby periodic orbits.[5]

For more general cases, the only guidance we have is the "overlapping criterion" by Chirikov. Almost all calculations made so far for beam-beam interaction rely on this to find the limiting nonlinear strength. Chirikov proposes that the density of stochastic layers is given by

$$\text{density} \sim s\, e^{-c/s}, \qquad c \sim 1$$

where s is the resonance width divided by the resonance separation calculated from the single-resonance model. When the density is of the order of unity, the entire phase space will become stochastic. It is certainly an order-of-magnitude criterion and any ambiguities regarding the resonance width (there are ambiguities) are not too important. Nevertheless, numerical studies made for a few examples have shown that the criterion is a surprisingly good one. Ruggiero and Smith, among others, applied the criterion to two-dimensional (one degree of freedom) beam-beam interaction[6] and found that the linear tune shift* ξ of the stochastic limit is ~ 0.25. Later, Ruggiero extended this to the four-dimensional case.[7] The limit is $0.125 \sim 0.25$ depending on the shape of the beam at the interaction point. In either case, the limiting value is large compared to what one usually encounters in storage rings. For example, ξ is at most 0.001 per intersection in the ISR (eight intersections) and < 0.06 in most electron storage rings. Incidentally, there is always a question of whether the relevant value of ξ should be

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* I regret I have to use this unfortunate jargon here.

per intersection or per revolution. Some data from electron storage rings show that one should take $\sqrt{n}\ \xi$ when there are n intersections in the ring. Although the data are qualitative rather than quantitative, they are suggestive of the involvement of some random processes.

What happens if the number of dimensions (or degrees of freedom) is more than two? It is quite possible (although I am not absolutely sure on this point) that the stochastic limit will be lower when more dimensions come into play. The number of relevant dimensions depends not only on added directions of motion but also on whether the parameters are static or time-varying, whether the momentum dispersion and chromaticity are zero and whether the collision is head-on or with an angle. It is certainly prudent to design the ring in such a way that the number of dimensions is kept as small as possible. However, I know of no work giving any quantitative estimates on this question. The KAM theorem stil assures us that, below stochastic limit, there will be KAM surfaces in phase space when there are more than one degree of freedom involved. Unfortunately the surface cannot surround a finite closed volume. For example, when there are two degrees of freedom (four-dimensional), KAM surfaces are all two-dimensional so that particles can in principle move out of one stochastic layer to another. Eventually, they may sneak out to a region of phase space far away from the center and this may cause a slow growth in beam size. This phenomenon is called the "Arnol'd diffusion" and it has been one of the most fashionable topics in accelerator physics for the past ten years or so. Even today, the condition $\xi \lesssim .005$ (the value at which the Arnol'd diffusion is supposed to become significant) is considered by some accelerator physicists to be something sacred. Actually, if one tries to trace the origin of this number, the search often ends up in some unpublished works or informal talks associated with names like Keil and Courant. According to Moser,[8] the Arnol'd diffusion has been clearly established for only one example. There are of course numerical studies which indicate a relatively fast growth in beam size when, for example, the strength of nonlinearity is modulated in time.[9] It is not clear to me whether they are genuine examples of the Arnol'd diffusion or simply vagaries of particular numerical simulations.

After this long and rambling story on an ambiguous subject, I am confident that anyone who has endured to this point is anxious to hear about the works by Kheifets and by Ruggiero.

III. Kheifets Model[2]

The idea that the interplay of diffusion, which exists in any storage rings, with the nonlinear beam-beam interaction must be a factor in determining the beam lifetime is a rather old one. One comment made by Hereward in 1972 has already been mentioned (see p. 4) as well as qualitative evidence from beam studies in electron storage rings (see p. 8, top). Already in 1969, Chirikov, Keil and Sessler[10] in their report on the stochastic limit of nonlinear oscillating systems had this to say:*

"We have not treated the joint influence of both non-linear resonances and a diffusion process of some kind, e. g. gas scattering. In this case, non-linear resonances may accelerate the diffusion process considerably even if without diffusion the motion is absolutely stable. It is not excluded that .... it might be of importance in the application of this work to proton storage rings."

I mention these not to downgrade the originality of the works by Kheifets and by Ruggiero but rather to stress that their ideas are eminently reasonable.

Kheifets restricts his work to 1) electron storage rings, 2) one degree of freedom (vertical motion only), and 3) weak-strong instability. The last condition implies that one beam is much stronger than the other so that the nonlinear field produced by the strong bunch is not affected by the beam-beam interaction.** A test particle in the weak bunch gets a nonlinear, delta-function kick (thin-lens approximation) at the interaction point. The amount of kick of course depends strongly

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* One might regard this simply as an example of "Keep-the-escape-hatch-open" style of writing.

** In practically all calculations on the beam-beam interaction in storage rings, this approximation is standard. In other areas such as low-energy proton or heavy-ion linacs and low-energy transport systems, the requirement for self-consistency is essential in space-charge problems.

on the position of the particle Y. Kheifets uses the variables $y \equiv Y/\sigma_v$ and $\dot{y} \equiv dy/dt$ where σ_v is the rms vertical size of the strong bunch.* Each time a particle passes through the strong bunch, it gets the kick

$$\Delta y = 0 \quad \text{and} \quad \Delta\dot{y} = F(y)$$

where F(y) is determined by the particle distribution of the strong bunch. The distribution function $f(t, y, \dot{y})$ of the weak bunch satisfies the Fokker-Planck type of diffusion equation which now contains the term

$$(n\omega_o/2\pi)\ \{f[t, y, \dot{y}+F(y)] - f[t, y, \dot{y}]\}$$

where n is the number of (identical) interaction points in the ring and $\omega_o/2\pi$ is the revolution frequency. The factor $(n\omega_o/2\pi)$ is just the number of nonlinear kicks per second. In writing this way, one is only interested in the beam size variation averaged over many revolutions. Since F(y) is small, one can expand the expression into a series in F(y). The crucial part of the model comes into the next step.

Kheifets argues that the strong nonlinearity of beam-beam interaction will lead to a "fast mixing" of particles within the weak bunch. The particle motion becomes random and, regardless of its initial coordinates, a particle is capable of appearing at any point in phase space occupied by the bunch. This statement is certainly true above the stochastic limit. Below the limit, however, there are KAM surfaces which are impenetrable and any randomness is mostly in the direction of phase and not in the direction of amplitude. Stochastic layers are extremely thin in the central area of phase space. For electron storage rings, there is of course quantum excitation in addition to gas scattering and intrabeam scattering. One can therefore imagine that the fast mixing is more likely in electron storage rings than in proton storage rings. Kheifets mentions these diffusion effects but he certainly considers the diffusion action of beam-beam interaction to be independent of any other mechanism. This is the point Ruggiero objects and I must agree with him. If one accepts the argument by Kheifets, F(y), $F^2(y)$, etc. must be averaged *over all particles in the weak bunch. Clearly the first term,* linear in F(y), vanishes for a symmetric (in y) beam. The second term when averaged over the distribution $f(t, y, \dot{y})$ gives the diffusion coeffici-

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* I do not always use the same symbols as Kheifets.

ent D_{bb}:

$$D_{bb} = (n\omega_o/4\pi) \langle F^2(y)\rangle$$

Fokker-Planck equation for f now contains this new diffusion term due to the beam-beam interaction. However, it is obvious that one should not include the linear part of F(y) for the estimate of D_{bb}. Effects of the linear part are entirely predictable and the stochastic behavior cannot be caused by the linear part. How to subtract the linear part is another feature of his model and Kheifets is forced to introduce an arbitrary recipe with one parameter which should be found by fitting experimental data to his formula. He says

> "It is not quite clear yet if and how this constant can be expressed through physical parameters of the storage ring."

Ruggiero's model is a step to rectify this situation.
In the limit $t \to \infty$, the solution of the Fokker-Planck equation gives the stationary beam size $\bar{\sigma}_v$,

$$\bar{\sigma}_v^{\,2} = \sigma_v^{\,2} + D_{bb}/(2\alpha)$$

where α ($\equiv$ 1/damping time) is the damping rate of the vertical oscillation. Since D_{bb} is an integral over the distribution of the weak bunch with $\bar{\sigma}_v$, this is an equation for $\bar{\sigma}_v$.

For electron beams, the vertical beam size σ_v is usually much smaller than the horizontal beam size σ_h. The beam-beam interaction F(y) is

$$F(y) = (2\pi c/\beta_v^*)\cdot\xi\cdot\sqrt{1+2b}\cdot\phi(y),$$

$$\phi(y) = -\,y\int_0^1 du\,\frac{1}{\sqrt{u+b^2}}\cdot e^{-uy^2}$$

where ξ is the linear tune shift, β_v^* is the betatron amplitude function at the interaction point and $b \simeq \sigma_v/\sigma_h \ll 1$. The function $\phi(y)$, which is antisymmetric in y, is shown in Fig. 3 for $b = 0.04$. If one subtracts the linear part of $\phi(y)$,

$$\text{linear part of } \phi(y) \simeq 2(1-b)\cdot y\,,$$

the resulting kick becomes unreasonably large at large values of y.

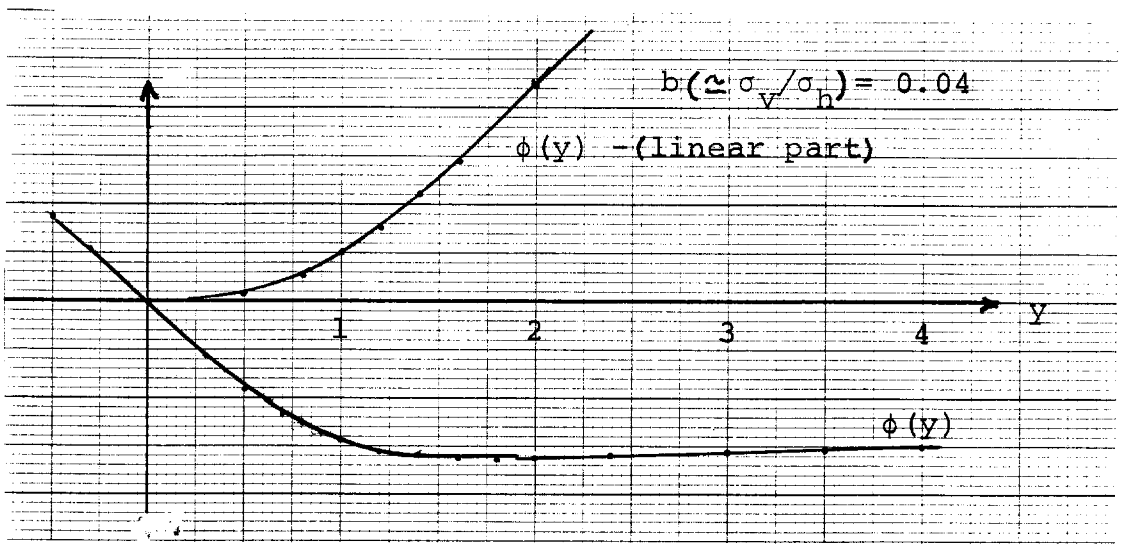

Fig. 3

Kheifets considered the introduction of some cut-off factor but this did not give a satisfactory answer. The subtraction procedure he finally arrived at is

$$\tilde{\phi}(y) \equiv \phi(y) - (1 - h)\cdot\phi\left(\frac{y}{1 - h}\right) , \quad 0 < h < 1$$

where h is the (yet unknown) parameter. The linear part of $\phi(y)$ is subtracted without introducing a large contribution at large y values. The diffusion coefficient D_{bb} is now

$$D_{bb} = (n\omega_o/4\pi)\cdot\langle F^2(y) \text{ with } \tilde{\phi}(y) \text{ instead of } \phi(y)\rangle .$$

One can see without too much difficulty that the blow-up factor of the beam size $d \equiv \bar{\sigma}_v/\sigma_v$ is given by the expression

$$d^2 = 1 + \eta\times(\text{average of } \tilde{\phi}^2(y) \text{ over the distribution of the weak bunch with } \bar{\sigma}_v)$$

where*

$$\eta = (n\omega_o/\alpha)\cdot\xi^2 .$$

Kheifets then calculates (numerically) d as a function of $\sqrt{\eta}$ for various combinations of b and h and plotts d vs $\sqrt{\eta}$. The best fit

* η in ref. 2 is this η times π.

for SPEAR data (b=0.035) has been found to give h = 0.04 (see Fig. 4).

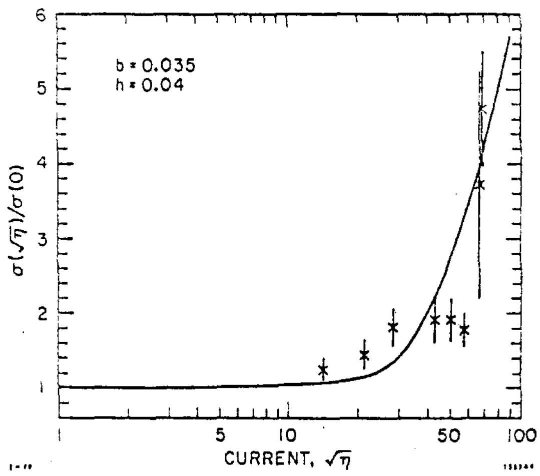

Fig. 4. The comparison of calculated beam blow up (the curve) with the measurements[5] of the vertical size of the weak beam (points). The strong bunch aspect ratio b = 0.035. The value of the fitting parameter h = 0.04. The bars represent the measurement errors only, and do not include any instrumentation errors.

A point he emphasizes at the end of his paper is that the proper parameter for the beam instability is <u>not</u> ξ , the linear tune shift, but η (or $\sqrt{\eta}$). The parameter $\sqrt{\eta}$ is proportional to the square root of the number of kicks per second and its energy dependence is $E^{-9/2}$ provided h is independent of these variables.

What really is the meaning of the subtraction procedure Heifets introduced more or less arbitrarily? If h=0, $\tilde{\phi}(y)=0$ and there will be no beam blow up. If h=1, $\tilde{\phi}(y)=\phi(y)$ and the entire beam-beam interaction (including the linear part) contributes to the stochastic mixing of particles (which of course is not the case). In this sense, the value of h is a measure of what fraction of the beam-beam interaction plays a role in the stochastic behavior of the beam. The particular way Kheifets does the subtraction is really much more specific than that. In Fig. 5, the "local gradient"

$$\tilde{\Phi}(y) \equiv \tilde{\phi}(y)/y$$

is plotted for h=1, 0.1, and 0.05 (enlarged by a factor ten) when b=0.04.

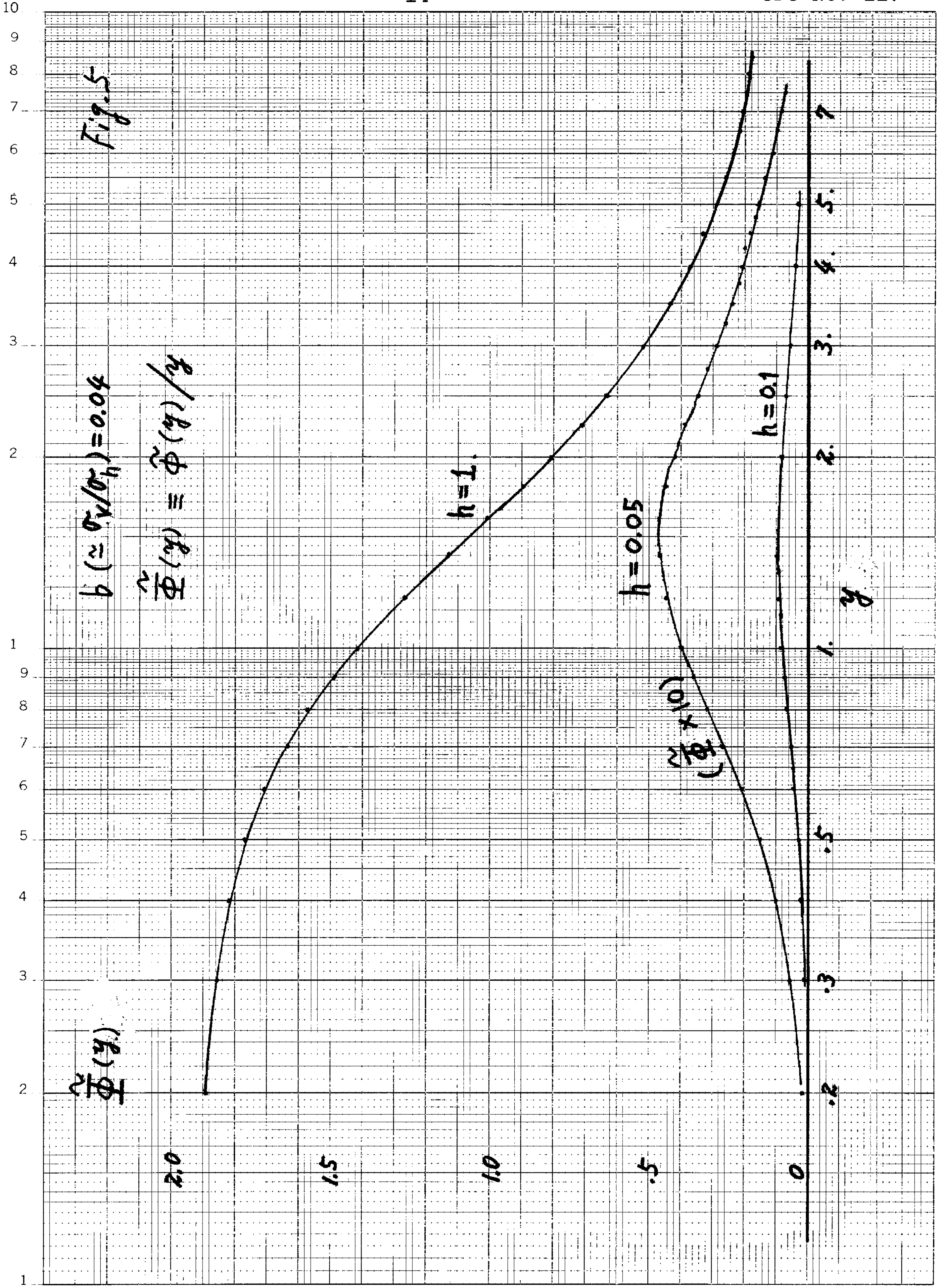
Fig. 5
b (≃ σv/σh) = 0.04
Φ̃(y) ≡ Φ̃(y)/y
h=1.
h=0.05
h=0.1
(Φ̃×10)
Φ̃(y)
y

Whether he intended it or not, his subtraction procedure seems to emphasize the beam-beam interaction around $Y = (1{\sim}2)\cdot\sigma_y$ as the one most responsible for the stochastic behavior of the beam and this is more or less independent of the value of h as long as it is less than ~ 0.1. I mention this fact (which is not in ref. 2) since it fits nicely with the model by Ruggiero. A more pronounced picture of this property will emerge if one plots the second derivative of $\mathfrak{F}(y)$ with respect to y.

For proton storage rings, α (damping rate) is for all practical purposes zeo. If applied directly to this case, Kheifets model gives a trivial result that the beam blow up is infinitely large since there is no stationary solution. As for the parameter η in place of ξ, as long as one accepts the concept of random kicks by the beam-beam interaction, it is rather natural to say without any more elaborate argument that the beam size will grow in time t more or less with $\sqrt{n\omega_0 \cdot t}\cdot\xi$ dependence, the square-root of the total number of kicks times the strength of the kicks. It is conceivable that one may gain more useful information on the beam lifetime by solving the Fokker-Planck equation. However, because of the total lack of understanding of the nature of parameter h, anything one says with this model about the beam lifetime of proton storage rings will be of rather speculative nature.

One minor (major?) defect of the model is that the tune of the machine does not play any role. In most electron storage rings, the dependence of the maximum achievable value of ξ on the tune is not at all insignificant. Electron beams seem to prefer the tune to be near integers. Neither is the model by Ruggiero free of this defect.

IV. Ruggiero Model*

Sandro may object to my use of the word "model" in describing his work. His intention, which is indeed admirable, is to make the maximum use of what we know mathematically about the nonlinear systems. The ultimate goal in this approach is to solve nonlinear problems complete-

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* The work discussed in this section is based on the talk which Ruggiero gave on October 15th. Any progress he may have made since then is therefore not covered.

ly - a feat worthy of at least a Heineman Prize.

Ruggiero starts by clasifying a storage ring (either for electrons or for protons) in four categories: 1) linear and quiet, 2) linear but noisy, 3) nonlinear but quiet, and 4) nonlinear and noisy which is the object of the study. Here "noisy" characterizes the existence of some diffusion mechanism such as irregular power supply ripples, RF noise, residual gas scattering, intrabeam scattering, and quantum emission (if electron storage rings). Nonlinear field could be either of magnets or of beam-beam interaction but the model is clearly meant for the latter. Again it is a case of weak-strong beam situation like Heifets model and it deals with one degree of freedom although there is no conceptual difficulties (technical difficulties, yes) to extend this model or the Kheifets model to two or more degrees of freedom.

Class 1. <u>linear and quiet</u>

The beam behavior is completely* solved by the Courant-Snyder work. One can define the "emittance" $\pi\varepsilon$ (or just ε) of a particle by writing the excursion $y(s)$ in the form

$$y(s) \equiv \sqrt{\varepsilon \cdot \beta(s)} \cdot \cos \psi(s)$$

where s is the distance along the closed orbit, $\beta(s)$ and $\psi(s)$ are, respectively, the amplitude and phase function of the betatron oscillation. The quantity ε is a constant of the motion, $d\varepsilon/dt = 0$.

Class 2. <u>linear but noisy</u>

Assume that there are a large number of small kicks which are totally random. After each kick, there is no change in y but $y' \equiv dy/ds$ will change by a small amount. Since each kick is uncorrelated, the net result is a diffusion,

$$d\varepsilon/dt = D_o$$

where the diffusion parameter D_o is specified by the given mechanism, e. g., gas scattering. If the ring is made of one or more identical

* On p. 11 of Courant-Snyder (Annals of Physics, <u>3</u>, 1 (1958)), we see the following statement: "On the boundary between stable and unstable regions ($|\cos \mu| = 1$) the treatment given here breaks down altogether." I am not aware of any work which clarifies this question.

periods and the revolution time is T per period, the expected emittance growth after n periods is

$$\langle \Delta\varepsilon \rangle_n = n \cdot T \cdot D_o$$

The corresponding changes in y and y' after n periods, which are called d_n and d'_n by Ruggiero, are given by the relations

$$\langle d_n \rangle = \langle d'_n \rangle = 0,$$

$$\langle d_n^2 \rangle = \beta_o nTD_o/2 \;, \qquad \langle d'^2_n \rangle = \gamma_o nTD_o/2$$

where (β_o, γ_o) are the Courant-Snyder betatron oscillation parameters at the point where (y, y') are observed. These are all familiar to any accelerator physicist who has ever computed the effect of random errors on the orbit whatever the origin of these errors may be.

Class 3. <u>nonlinear but quiet</u>

This is essentially the situation discussed in Section II. The effect of a kick by the beam-beam interaction is again

$$\Delta y = 0 \quad \text{and} \quad \Delta y' = \xi \cdot F(y) \;.$$

For a head-on collision of two round beams with the rms beam size σ, the shape of F(y) is shown in Fig. 6. This would be the case for our $\bar{p}p$ collider. Since we are still far below the stochastic limit ($\xi \ll 0.25$), it is still meaningful to talk about the more or less invariant emittance ε and the tune of a particle can be considered as a function of its emittance ε or of the amplitude $\sqrt{\beta^* \cdot \varepsilon}$ at the interaction point.

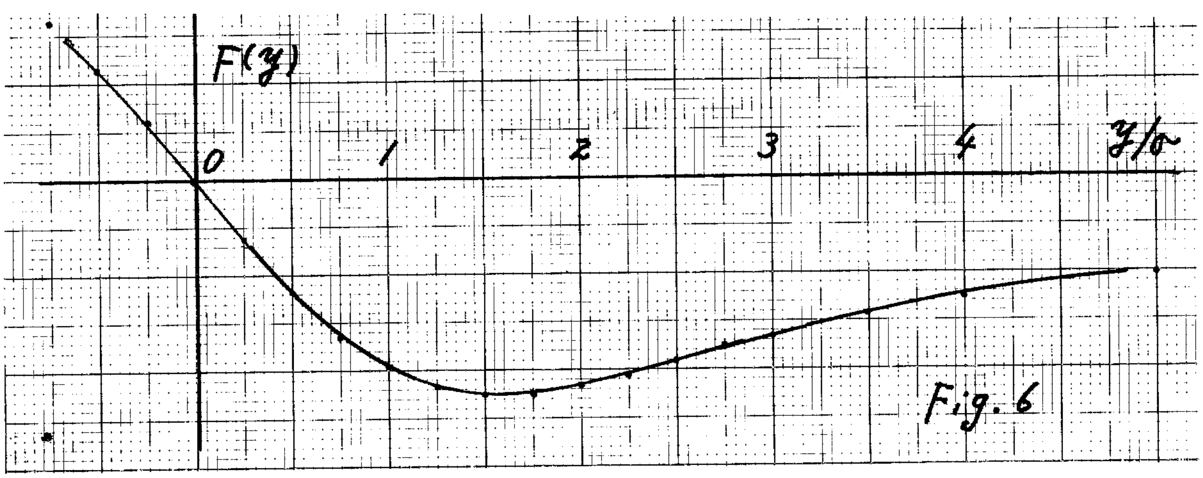


Fig. 6

One would naturally choose the unperturbed tune ν_o (which is the tune for all particles in the absence of nonlinear field) such that the perturbed tune of any part of the beam, $\nu_o < \nu < \nu_o + |\Delta\nu|_{max}$ does not hit low-order (order less than $\sim$10) resonances. Ruggiero argues that, under this condition, because of the characteristics of $\Delta\nu$ shown in Fig. 7 schematically, relatively low-order resonances will exist mostly around the amplitude $\sqrt{\beta^* \cdot \varepsilon} \simeq (1 \sim 2)\sigma$.

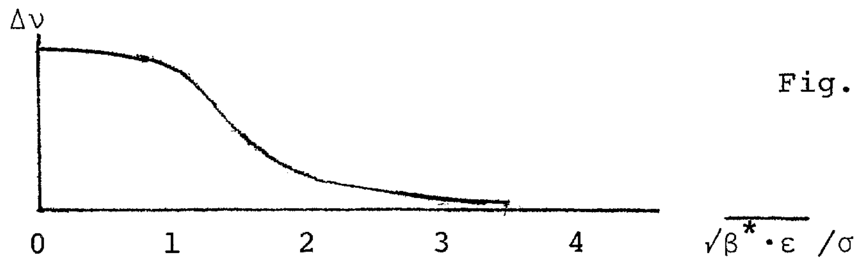


Fig. 7

Note that, mathematically speaking, resonances are everywhere dense but the associated stochastic layer is extremely thin unless the resonance is of low order. According to Ruggiero, stochastic layers that will become responsible for the diffusion-like growth of the beam size are mostly near $(1 \sim 2)\sigma$. The picture seems to be a justification or a physical interpretation of the subtraction procedure introduced by Kheifets.

Class 4. <u>nonlinear and noisy</u>

The first thing Ruggiero does for this case is to split y into two parts:

$$y = \bar{y} + \delta$$

where δ is the contribution from noise (all types of diffusions other than the beam-beam interaction) and $\bar{y}$ is what one would have if the ring is quiet but nonlinear (Class 3). For $\xi < \xi_{limit}$ (below the stochastic limit), one can still assign a more or less invariant emittance ε to each particle,

$$\bar{y} \simeq \sqrt{\beta^* \cdot \varepsilon} \cos \psi .$$

With small δ,

$$\Delta y' = \xi \cdot F(\bar{y}) + \xi \cdot F'(\bar{y}) \cdot \delta + \cdots\cdots$$

The effect of the first term has already been treated in Class 3 and, Ruggiero argues, this cannot lead to a diffusion-like growth of the beam size. One is then left with the second term where there is no correlation between $\bar{y}$ and δ. The crucial assumption (the word used by Ruggiero) is now introduced. The kick $\Delta y'$ can be treated as a random kick, which contributes to the growth of beam size, if and only if $\bar{y}$ is in a stochastic layer. The proper expression for $\Delta y'$ is then

$$\Delta y' = P(\varepsilon) \cdot \xi \cdot F'(\bar{y}) \cdot \delta$$

where $P(\varepsilon)$ is the probability for the particle with emittance ε to be in a stochastic layer. The estimate of $P(\varepsilon)$ is of course the most difficult part of this model.

Perhaps this is the right place to examine the essential difference of Ruggiero's model from Kheifets model. Kheifets assumed that $\Delta y'$ from the beam-beam interaction can by itself be the cause of a diffusion existing even when all regular diffusion mechanism is absent. In Ruggiero's model, the existence of regular diffusions (called "noise" by him) is absolutely essential for the beam-beam interaction to be a cause of additional diffusion. If one regards the parameter h to be independent of machine conditions, Kheifets model will still give the universal limit, to be sure in η and not in ξ, of the beam-beam interaction strength. The limiting value must apply to the ISR, the SPEAR, the doubler or any other storage rings. In contrast with this, the model by Ruggiero emphasizes the importance of environment in each ring. If ring A is noisier than ring B, the limiting strength of beam-beam interaction for A will be lower than for B. In the limit of no noise, there will be no beam size growth as long as one stays below the stochastic limit. The arbitrary subtraction procedure in Kheifets model is of course quite different from the assumption in Ruggiero's model where the concept of stochastic layers and the randomness of phase within the layers play the essential role.

Once his assumption is accepted, the rest is straightforward, only questions one may have being of a purely technical nature and consequently of no general interest. After n periods,

$$\delta_n = d_n + \xi \cdot \beta^* \cdot \sum_{p=1}^{n-1} G_{n,p} \cdot \delta_p$$

where the first term is the effect of regular diffusions (see Class 2) and the second term is the accumulation of effects coming from all previous interaction points (considered to be identical); β^* is the betatron amplitude function at the interaction point. $G_{n,p}$ is a function of $P(\varepsilon_p)$, $F'(\bar{y}_p)$ and the phase advance μ from one interaction point to the next. If one is interested only in the onset of the effect of beam-beam interaction (threshold value), the second term can be considered small compared to the first term,

$$\delta_n \simeq \sum_{p=1}^{n} a_{n,p}\cdot d_p \quad \text{with} \quad a_{nn} = 1,$$

$$a_{n,p} = \xi\beta^* P(\varepsilon_p) F'(\bar{y}_p)\sin(n-p)\mu \ .$$

Since the phase is completely random in a stochastic layer, one defines the average effect with the function $g(\varepsilon)$,

$$g(\varepsilon_p) = (1/2\pi)\int_0^{2\pi}[F'(\bar{y}_p)]^2 \cdot d\psi \ , \quad \bar{y}_p \simeq \sqrt{\beta^*\varepsilon_p} \cdot \cos\psi$$

The total diffusion is described by a new parameter D_n defined by

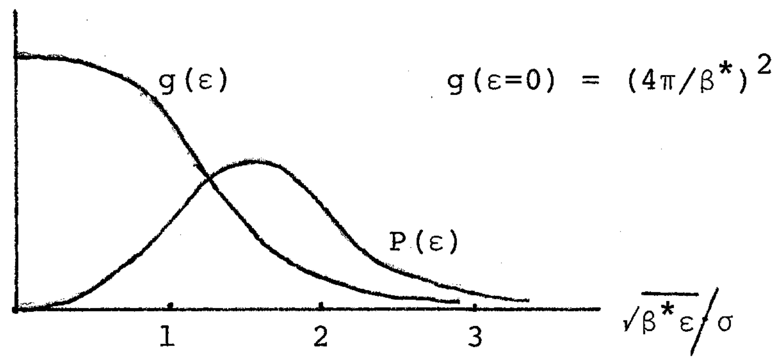
$$D_n = (2/\beta^* T) \ [\ <\delta^2_{n+1}> - <\delta^2_n> \ ] \ .$$

This of course is proportional to the original diffusion parameter D_o. Again I emphasize the difference from Kheifets model where D_n should be $D_o + D_{bb}$ and D_{bb} is totally independent of D_o.

The variation of the emittance ε as a function of time t is, for m identical interaction points in the ring,

$$d\varepsilon/dt \simeq D_o + (D_o/2)\cdot t\cdot(m\omega_o/2\pi)\cdot\xi^2\beta^{*2}\cdot(P^2 g)$$

where $(m\omega_o/2\pi)$ is the number of interaction per second. The quantity $P^2 g$ should be considered as the average of $P^2(\varepsilon)g(\varepsilon)$ when ε is changing. Since one is interested only in the threshold criterion, this point should not be too important. The threshold criterion would be either the second term itself small or the second term much smaller than the first term depending on how one looks at the problem. One would use the desired lifetime for t to find the safe value of ξ or $\sqrt{D_o}\cdot\xi$. For the head-on collision, Ruggiero gives the following picture.



Ruggiero looks at electron storage rings with this model but I will omit the discussion since there is no new ideas introduced in it.

V. What I Think

After laboring for a report of this size, my mind usually becomes all blurred, not an ideal condition for making a profound observation. On the other hand, there is no guarantee that a clear mind would produce better ideas on topics like this.

Works by Kheifets and by Ruggiero are certainly very interesting. They are both original in that the cause of beam size growth is sought in the area which has never been studied quantitatively. Of the two works, the one by Ruggiero is more satisfying to me but at the same time more difficult to apply. The difficulty lies in the quantitative evaluation of $P(\varepsilon)$, the probability for a particle of emittance ε to be in stochastic layers. Ruggiero suggests using the expression by Chirikov (see p. 7),

$$P(\varepsilon) \sim s(\varepsilon)\cdot e^{-c/s(\varepsilon)} \quad , \quad c \sim 1 .$$

The trouble is that, for a small value of $s(\varepsilon)$, the precise value of c becomes important to make a quantitative evaluation of P. As for the model by Kheifets, it is hard for me to imagine that the parameter h is universal to any storage rings, electrons and protons. The particular value he obtained from the SPEAR data, 0.04, cannot be right for the doubler.

It is fortunate that we have at least one accelerator physicist active in this field. We should probably be more attentive to what Sandro Ruggiero says.* We should certainly be alert to developments at other places, especially at CERN where a series of beam studies are planned in the ISR. It is a conventional wisdom to say that we should have a complete control of the chromaticity, the dispersion at the interaction point, the power supply ripples and the rf noise. In the absence of any unconventional wisdom, I believe it is worth repeating here.

* Whether we should follow all his advices is of course an entirely different matter.

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