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THE QUADRUPOLE COMPONENT IN DIPOLES AND THE INJECTION MISMATCH

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I. Introduction

By now, it has been well established[1] that shimming the cryostat off-center in yoke to remove quadrupole components in a dipole leads to substantial forces acting on the coil. As a result, the quadrupole components may depend strongly on the excitation current, certainly an undesirable feature for running the doubler. S. Wolff has suggested shimming the key angles on the collars as the only reasonable way to reduce the quadrupole components. Although this may turn out to be quite practical and successful in maintaining the present (rather stringent) criteria for quadrupole components,*

$$|b_1| \ , \ |a_1| \ < \ 2.5\times10^{-4}/\text{inch},$$

it is necessary, I believe, at this time to do some soul-searching re-evaluations of the criteria. Practical benefits one gains by relaxing the tolerance by a factor of two, for example, are so obvious that one is tempted to do so without examining its consequences. However, even without doing any detailed calculations, we may at the least have to plan for an expanded quadrupole correction system of the 39th harmonic component.[2] On the other hand, regardless of what new criteria we take, it is unlikely that the tune perturbation caused by the average quadrupole component becomes large enough to require an upgrading of the trim

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* On the median plane $y = 0$, $B_y(x) = B_o(1 + b_1x + b_2x^2 + \cdots)$ and $B_x(x) = B_o(a_1x + a_2x^2 + \cdots)$.

criterion for b_1, we may need more trim quadrupoles for this purpose than anticipated in the present design. With $(b_1)_{rms} < 3\times10^{-4}$/inch, there should be at least six at horizontal stations and six at vertical stations.

There is a potentially important side effect which is peculiar to the doubler when β and α are different from their ideal values. The doubler lattice has been designed by Tom Collins such that a single quadrupole makes the beam shape matched almost perfectly in both horizontal and vertical directions when the beam is transfered from the main ring to the doubler.[5] This is essential in confining the beam transfer systems, normal and reverse, in one long straight section E0. For the ideal case with the tune of 19.4 in both rings, we have, at the center of doubler long straight E0,

	β_x(m)	α_x	β_y(m)	α_y
MR beam	72.1	-.716	73.6	.734
Doubler beam	72.6	-.467	72.45	.466

Without any matching quadrupole, there will be an emittance dilution factor of 1.3 in each direction. With one quadrupole of modest strength ($B'\ell$ = 18 kG), the dilution can be reduced to less than a few percent simultaneously in both directions. Again it must be emphasized that, for the doubler, the beam emittance is a particularly important parameter. Superconducting magnets may be quenched by an otherwise insignificant beam halo. Since effects of high-order resonances are strongly dependent on the beam emittance, the beam lifetime of the collider will certainly deteriorate with the emittance dilution, this in addition to the obvious reduction in the luminosity.

In section II, expressions for $\Delta\beta/\beta$ and $\Delta\alpha$ are given. Numerical examples are presented in section III where the importance of knowing the real values of β and α at main ring E0 is discussed. Finally, a set of recommendations related to the subject of this note are given in section IV with the hope that they should be useful for further discussions on this matter in the doubler design group.

II. $(\Delta\beta/\beta)$ and $(\Delta\alpha)$ from Quadrupole Component b_1 in Dipoles

Derivation of expressions for $(\Delta\beta/\beta)$ and $(\Delta\alpha)$ can be found in the classical work of Courant and Snyder.[6] If the normalized phase ϕ, which is the betatron phase advance divided by the tune, is taken to be zero at the point where one is interested in evaluating the changes in β and α, we have, in the lowest-order approximation,

$$(\Delta\beta/\beta) = -\frac{1}{2\sin(2\pi\nu)} \sum_i k_i\beta_i \cos(2\pi\nu - 2\nu\phi_i),$$

$$(\Delta\alpha) = -\frac{1}{2\sin(2\pi\nu)} \sum_i k_i\beta_i \sin(2\pi\nu - 2\nu\phi_i) + \alpha(\Delta\beta/\beta)$$

where the summation is for all 774 dipoles and

ν, β, α = unperturbed machine parameters,

k_i = integrated quadrupole gradient divided by $(B\rho)$

$= (2\pi/774)\cdot b_1$ in each dipole.

In the real ring, there will be an error in each quadrupole also and its contribution to $(\Delta\beta/\beta)$ or $(\Delta\alpha)$ can be obtained by substituting

$$k_i = (\Delta\int B'ds)/(B\rho), \qquad \sum_i \text{ for all quadrupoles}$$

in the above expressions. An equivalent expression for $(\Delta\beta/\beta)$ is

$$(\Delta\beta/\beta)_\phi = -\frac{\nu}{\pi} \sum_n \frac{J_n}{4\nu^2 - n^2} e^{in\phi} \quad , \quad n = 0, \pm1, \pm2, \ldots.$$

with

$$J_n = \sum_i k_i\beta_i\, e^{-in\phi_i} = (2\pi/774) \sum_i (b_1)_i\beta_i\, e^{-in\phi_i}$$

This shows that the dominant contribution to $(\Delta\beta/\beta)$ comes from the harmonic components of $(b_1\beta)$ whose order is nearest to 2ν, the 39th harmonic for the doubler with $\nu = 19.4$. The corresponding expression for $\Delta\alpha$

is

$$(\Delta\alpha)_\phi = \alpha(\Delta\beta/\beta) + \frac{i}{2\pi} \sum_n \frac{nJ_n}{4\nu^2 - n^2} e^{in\phi}$$

which can be obtained from the relation

$$\alpha \equiv -\frac{1}{2}\frac{1}{\nu\beta}\frac{d\beta}{d\phi} .$$

In terms of the rms value of b_1,

$$(\Delta\beta/\beta)_{rms} = \frac{1}{2\sqrt{2}} \frac{1}{|\sin(2\pi\nu)|} \left(\frac{2\pi}{774}\right) \left(\sum_i \beta_i^2\right)^{1/2} (b_1)_{rms},$$

$$(\Delta\alpha)_{rms} = (1 + \alpha^2)^{1/2} (\Delta\beta/\beta)_{rms} .$$

For the standard doubler lattice with $\nu = 19.4$,

$$\left(\sum_i \beta_i^2\right)^{1/2} = 1.71\times10^3 \text{ m} \quad \text{horizontal}$$
$$= 1.73\times10^3 \text{ m} \quad \text{vertical}$$
$$(1 + \alpha^2)^{1/2} = 1.10 \quad \text{at E0, horizontal and vertical.}$$

With these values, we find

$$(\Delta\beta/\beta)_{rms} = 3.28\% \times (b_1)_{rms} \text{ in } 10^{-4}\text{/inch, horizontal}$$
$$= 3.33\% \times (b_1)_{rms} \text{ in } 10^{-4}\text{/inch, vertical}$$
$$(\Delta\alpha)_{rms} = 0.036 \times (b_1)_{rms} \text{ in } 10^{-4}\text{/inch, horizontal}$$
$$= 0.037 \times (b_1)_{rms} \text{ in } 10^{-4}\text{/inch, vertical}$$

III. Numerical Examples

When a beam with the Twiss parameters $(\beta_1, \alpha_1, \gamma_1)$ is injected from the main ring into the doubler ring at a place where the machine parameters are $(\beta_2, \alpha_2, \gamma_2)$, the emittance dilution factor is[7]

$$D + (D^2 - 1)^{1/2}$$

with $$D = (\gamma_1\beta_2 + \gamma_2\beta_1 - 2\alpha_1\alpha_2)/2 \; .$$

The emittance dilution factor in the four-dimensional phase space is defined here as the product of two factors, one in the horizontal and the other in the vertical phase space. In the following numerical examples, 500 cases with $\sigma \equiv$ standard deviation of b_1 = (1.5, 2., 2.5, 3.)$\times 10^{-4}$/inch and the cut-off of $|b_1| < 2\sigma$ have been used to calculate the minimum emittance dilution factor that can be obtained with a single matching quadrupole. In general, the dilution is less than its average value for 55% of the 500 cases and it is less than (average + rms) for 85% of all cases. Changes in the momentum dispersion parameters* X_p and $X_p' \equiv dX_p/ds$ are also given although they are not of immediate concern here.

A) <u>No errors in β and α in the main ring</u>

σ (10^{-4}/inch)	$(\beta_x)_{av}$	$(\alpha_x)_{av}$	$(\beta_y)_{av}$	$(\alpha_y)_{av}$	$(X_p)_{av}$	$(X_p')_{av}$
0	72.6m	-.4667	72.45m	.4661	2.43m	.0206
1.5	72.7	-.465	72.4	.467	2.43	.0205
2.0	72.8	-.465	72.5	.468	2.42	.0205
2.5	72.9	-.465	72.6	.469	2.42	.0205
3.0	73.1	-.466	72.7	.470	2.42	.0205

σ	$(\beta_x)_{rms}$	$(\alpha_x)_{rms}$	$(\beta_y)_{rms}$	$(\alpha_y)_{rms}$	$(X_p)_{rms}$	$(X_p')_{rms}$
1.5	3.21m	.049	3.29m	.050	.094m	.0014
2.0	4.29	.066	4.41	.067	.125	.0019
2.5	5.38	.082	5.54	.084	.157	.0023
3.0	6.49	.100	6.70	.102	.188	.0028

* The horizontal excursion x_p arising from the momentum deviation $(\Delta p/p)$ of a particle is defined to be $X_p(\Delta p/p)$.

Four-dimensional emittance dilution factor

σ	average	rms	maximum
1.5×10^{-4}/inch	1.12	0.06	1.38
2.0	1.17	0.085	1.54
2.5	1.215	0.11	1.72
3.0	1.26	0.14	1.94

B) The main ring is known to be far from ideal. There have been some measurements of β at a few places in the ring[8] and the results obtained so far indicate the error in β to be of the order of 10%. The plan to measure β and α in the long straight A in order to have a better understanding of the injection matching has been in existence for the past several years[9] but the main ring group simply could not find time to carry out the project. It is not inconceivable that, sooner or later, we may have to measure β and α at E0 of the main ring. If the errors there happened to be substantial, the emittance dilution introduced during the beam transfer would not be too sensitive to the amount of quadrupole component b_1 in doubler dipoles. In order to see this quantitatively, 1,000 cases with the uniform distribution in $(\Delta\beta/\beta)$ and $(\Delta\alpha)$ have been used to calculate the emittance dilution. The quadrupole components in doubler dipoles are assumed to be the same as for A),

$\sigma \equiv$ standard deviation of $b_1 = (1.5,\ 2.,\ 2.5,\ 3.)\times10^{-4}$/inch,

$|b_1| < 2\sigma$ cut-off.

1. $|\Delta\beta/\beta|_{max} = 5\%$ and $|\Delta\alpha|_{max} = 0.06$ at MR E0

	emittance dilution factor		
σ	average	rms	maximum
1.5×10^{-4}/inch	1.14	.066	1.49
2.0	1.185	.083	1.60
2.5	1.23	.103	1.74
3.0	1.275	.124	1.88

2. $|\Delta\beta/\beta|_{max} = 10\%$ and $|\Delta\alpha|_{max} = 0.12$ at MR E0

σ	average	rms	maximum
1.5	1.20	.100	1.72
2.0	1.23	.117	1.87
2.5	1.27	.135	2.02
3.0	1.31	.155	2.18

3. $|\Delta\beta/\beta|_{max} = 15\%$ and $|\Delta\alpha|_{max} = 0.185$ at MR E0

σ	average	rms	maximum
1.5	1.285	.142	2.00
2.0	1.31	.158	2.17
2.5	1.34	.177	2.35
3.0	1.37	.198	2.54

4. $|\Delta\beta/\beta|_{max} = 20\%$ and $|\Delta\alpha|_{max} = 0.25$ at MR E0

σ	average	rms	maximum
1.5	1.38	.195	2.33
2.0	1.40	.210	2.52
2.5	1.42	.229	2.73
3.0	1.45	.251	2.95

C) Improvements Expected from 39th Harmonic Correction Systems

With relatively small number of trim quadrupoles, it should be possible to eliminate the most harmful 39th harmonic component of βb_1 without at the same time increasing the amount of other harmonics. In order to simulate the effect of such correction systems, the same 500 cases as in A) with $\sigma = 3\times10^{-4}$/inch have been used but, this time, eliminating the 39th harmonic in each case.

four-dimensional emittance dilution

	average	rms	maximum
no correction	1.26	.142	1.94
39th harmonic eliminated	1.09	.043	1.26

With the present design value of $B'\ell = 60$ kG each, the correction of 39th harmonic will require at least twelve (and preferably twenty-four) trim quadrupoles divided into four independent sets if $\sigma = 3\times10^{-4}$/inch is adopted as the criterion for b_1.

IV. Recommendations

It is of course not possible to draw a sharp line to define the boundary between the "safe" and the "dangerous" values of b_1. One should naturally try to minimize the emittance dilution as much as possible but this must be done with a careful examination of the price one must pay now and in the future. The purpose of this note is not to make definitive and inflexible conclusions but rather to present a set of recommendations that should be discussed by the doubler group.

Recommendations

1. New criteria for the normal quadrupole component b_1 in doubler dipoles:

   660A to 4000A

   $$(b_1)_{rms} < 3\times10^{-4}/\text{inch}$$
   $$|(b_1)_{av}| < 1\times10^{-4}/\text{inch}$$
   $$|b_1| < 6\times10^{-4}/\text{inch}$$

2. There should be a plan to have at least twelve harmonic trim quadrupoles with four "knobs". If the price is not prohibitive, it is desirable to consider a system with twenty-four trim quadrupoles.

3. There should be a joint effort with the main ring group to measure β and α (both horizontal and vertical) at E0 in the main ring at 150 GeV/c. Depending on the result of this measurement, we may have to re-evaluate the plan for the harmonic correction system.

References

1. S. Ohnuma, memo of January 24, 1980; S. Wolff, memo of February 13, 1980.
2. This has been intimated in the design report issued in May, 1979. See section 7. 3. 2., p. 124.
3. S. Ohnuma, UPC No. 116, November 19, 1979.
4. S. Ohnuma, TM-910, October 15, 1979.
5. Design Report of the Superconducting Accelerator, May 1979, p. 176.
6. E. D. Courant and H. S. Snyder, Annals of Physics, 3, 1 (1958).
7. C. Bovet, et al., "A Selection of Formulae and Data Useful for the Design of A. G. Synchrotrons", CERN/MPS-SI/Int. DL/70-4, 23 April, 1970, p. 20.
8. EXP-97 (June 5, 1979), EXP-98 (August 27, 1979), EXP-98A (September 5, 1979).
9. S. Ohnuma, TM-736, June 8, 1977.