Fermilab

UPC 128

LONGITUDINAL INSTABILITIES REVISITED*

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I.

According to the scenario developed by the Colliding Beams Department for the $\bar{p}p$ collision in the doubler, there will be a dozen or so bunches of protons and these bunches are expected to contain at least 10^{11} particles each which is approximately four times the presently available bunch intensity in the main ring. Clearly, many bunches must be combined to form such an intense single bunch and, in view of the rather limited momentum acceptance of the doubler,[1] it seems essential that the dilution of the longitudinal beam emittance be kept as small as possible. The requirement of a relatively small number of intense proton bunches applies to the CERN $\bar{p}p$ collision in their SPS also and a series of accelerator experiments have been done there to study the acceleration and the storage of 10^{11} protons in one bunch at 210 GeV/c.[2] They believe that there are two longitudinal phenomena which are significant for the beam lifetime. The claim seems to be well justified by the fact that the lifetime of the intense bunch was increased from the initial few minutes to about eight hours by controling the two effects. The purpose of this note is to explain their current interpretation of the longitudinal beam instabilities and to predict what is likely to happen in the doubler if their interpretation is the right one for us as well.

The beam storage experiments at Fermilab[3] were performed mostly in 1976 and 1977, and again in 1979. Recent beam studies by the Colliding Beams Department have revealed many interesting and impor-

*It seems as though it is part of my fate to follow the footsteps of Sandro Ruggiero who wrote several excellent reports on the same topics and who is almost totally responsible for arousing my interest in this subject, one about which I knew very little when he volunteered to educate me four or five years ago.

tant phenomena but results are not yet available as a report.[4] From the earliest time, Jim Griffin and Sandro Ruggiero have contributed most to our understanding of the longitudinal beam behaviors observed during storage experiments. Based on this experience and also on his works, Ruggiero wrote several reports on the subject of longitudinal instabilities expected in the doubler and proposed a number of recommendations.[5] His conclusions are still basically valid and in an essential agreement with the CERN interpretation.[6] The present note should not therefore be regarded as one to replace Ruggiero's work but rather as one complementary to it.

II.

Very high-Q, narrow-band resonators like RF cavities are responsible for the coupled-bunch instabilities because of their long wake field. In most cases, these coupled modes are sufficiently suppressed and one can disregard them in the discussion of longitudinal behavior of a stored beam. This is particularly true when a few bunches in the ring are well separated. The ring with its bellows, detectors and discontinuities in the vacuum chamber cross section is then believed to be equivalent to a low-Q, broad-band resonator as far as the effect on the beam is concerned. Up to $\sim$ 1 GHz, which corresponds to the bunch spectrum in most proton machines, the real part (resistive) and the imaginary part (positive, i. e., inductive) of the coupling impedance Z are more or less of the same magnitude and they are proportional to the frequency,

$$|Z/n| \simeq \text{constant}, \quad n \equiv \text{freq/rev. freq.}$$

Around $\sim$ 1 GHz, Z is mostly resistive since the sign of the imaginary part changes from positive (inductive) to negative (capacitive). The picture given here is admittedly a rather simple-minded one but the result on the beam instability seems to be fairly insensitive to the details of how Z behaves as a function of frequency.

According to the CERN interpretation, there are three mechanisms by which the longitudinal beam emittance can grow and may even cause a beam loss. In the order of their expected appearance in time during a storage, they are: 1) microwave instability, 2) coherent modes instability, and 3) RF noise. Effects of the first and the second may last up to ∿ 30 minutes and the last mechanism may cause a slow diffusion of particles out of the RF bucket. It must be pointed out here that this interpretation probably does not apply in all of its details to our experiences in the main ring.* As will be explained below, the relative importance of these mechanisms can be very different from one machine to another. Besides, the bunch intensity in our storage experiments has usually been $(0.5 \sim 2)\times10^{10}$ instead of 10^{11} in the SPS.

<u>microwave instability</u>

This was discovered in 1974 by D. Boussard[7] in the CERN PS and since then has been extensively discussed by many people.[8] It is a very fast instability, developing in a fraction of a synchrotron oscillation period. It is therefore possible to treat the problem in the same manner as a coasting beam. Since the frequency involved is very high, from a few hundred MHz to ∿ 2 GHz in the SPS, for example, it may be reasonable to assume that the responsible impedance is primarily resistive.[9] However, it is a more common practice to describe the instability in terms of an equivalent $|Z/n|$ and the figure quoted for the CERN PS, SPS and our main ring[10] is always $(20 \sim 30)\Omega$. This value together with the beam parameters indicate that, for us as well as for SPS, the microwave instability is the least important one of three mechanisms in spite of its once very fashionable status.

<u>coherent modes instability</u>

This instability is expected to be very slow, typically developing over many synchrotron oscillation periods. For example, the quadrupole mode oscillation in the SPS has been found to develope over $>10^4$ oscillations under certain conditions of the bunch. According to the classical work by Frank Sacherer on the longitudi-

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* This has been emphasized by Jim Griffin who undoubtedly has observed more stored bunches in the main ring than anyone else.

nal stability of bunched beams,[11] the imagninary part of coherent frequency shift $\Delta\omega_m$, $\mathrm{Im}(\Delta\omega_m)$, of the single-bunch mode (n = 0 in his notation) is always zero or negative above transition. Here, m is the mode number, 1 for dipole mode, 2 for quadrupole mode, etc. Since the amplitude growth rate of the mode is given by $\mathrm{Im}(\Delta\omega_m)$, the mode is in principle never unstable. However, the large $|\Delta\omega_m|$ suppresses the Landau damping since it may exceed the amount of the spread in the synchrotron oscillation frequency within the bunch. Under such condition, it is possible for the bunch to experience weak instabilities by some mechanism. A possible cause of this has been mentioned by Sacherer (p. 827 of ref. 11, under Fig. 5) who later proposed another possibility of instability for a very high intensity bunch.[12] CERN people observed that, when the instability is fully developed, the motion is often very complicated with many modes existing simultaneously. One must of course remember here that this instability may depend strongly on the details of beam environment and the observation in the CERN SPS might not apply to the main ring even with the same beam parameters.

Looking back , one cannot help wondering if the word "microwave instability" has been invoked indiscriminately to describe any kind of single-bunch longitudinal instabilities.[9,11] Perhaps the true microwave instability as observed originally by Boussard is important in the SPS and in the main ring only under a very special condition, for example a very high intensity bunch with a small longitudinal emittance* or during a beam debunching.[8] The microwave instability during a debunching of more than one bunch is a very difficult problem because of the complication of bunch overlapping (multi-stream instability).

RF noise

When the bunch length becomes sufficiently large because of the coherent motion, the nonlinearity of the RF bucket re-establishes the spread of synchrotron oscillation frequency within the bunch and the resulting Landau damping stabilizes the bunch motion. At the same time,

* This seems to be the case when the injected bunch from the CPS into the SPS at 26 GeV/c keeps its low emittance. CERN people are talking about blowing up the emittance by a factor 3 (refs. 6 and 13) in order to prevent the possible microwave instability.

this large bunch is believed to be particularly sensitive to RF noise and particles can diffuse out of the RF bucket. Jim Griffin believes that, in the main ring, RF noise and other noises related indirectly to RF may be playing an important role from the beginning of beam storage, coexisting with the coherent mode oscillations but not necessarily contributing to the beam loss. The existing theory[14] shows that the diffusion rate $d\langle A^2\rangle/dt$ (where A is the phase oscillation amplitude) is proportional to S^2G_e,

S = synchrotron oscillation frequency spread within the bunch,
G_e = spectral density of the noise voltage which is assumed to be a white noise.

Experiments in the ISR and in the SPS seem to confirm that this theory can explain the diffusion in its various aspects. In the ISR,[14] S was increased by the higher harmonic (Landau) cavity while in the SPS, G_e was reduced by changing the radial loop bandwidth from 1 kHz to 10 Hz.[2]

I am not really qualified to discuss the problem associated with RF noise. Perhaps Jim Griffin and others will write reports on what happens in the main ring because of the noise in general. I will just mention here that the phenomenon must be quite different in different machines. For example, in the SPS, the phase amplitude diffusion $d\langle A^2\rangle/dt$ is considered to be equivalent to the beam lifetime. This is reasonable since the equilibrium bunch length ($\sim$ 3 ns) is not much shorter than the full bucket length of 5 ns. In the main ring, it is inconceivable that, even with the intensity of 10^{11}/bunch, the bunch length will be $\sim$60% or more of the bucket length which is 19 ns. A question arises as to whether, in the main ring, we would experience serious effects of RF noise which would cause a beam loss if the spread S were increased by a Landau cavity (for example) in order to suppress the coherent motion. It has been shown[8] that one can maintain the bunch length or even decrease it with the Landau cavity if the beam area is large and the harmonic number of the Landau cavity is high. In any case, one thing is quite certain: we do need more beam storage experiments in the main ring in order to sort out various longitudinal phenomena so that it would be possible to make an intelligent plan for the doubler.

III.

The doubler design report[15] assumes that the longitudinal beam emittance at 150 GeV/c (injection) is 0.25 eV-s when the beam intensity is 2×10^{10}/bunch. During the debunching and the rebunching to increase the bunch intensity to 10^{11}/bunch, there will be a sizable amount of emittance dilution. Additional dilution must be expected during the beam transfer from the main ring to the doubler and the acceleration in the doubler from 150 GeV/c to 1 TeV/c. Therefore it seems reasonable to assume at this time that, for the $\bar{p}p$ collision, the longitudinal beam emittance of the proton bunch will be $\gtrsim$ 2 eV-s.[16] The peak RF voltage in the doubler with six cavities[17] is approximately 2 MV/turn which is large enough to provide a stationary bucket of the area 6 eV-s at 150 GeV/c. For stationary bucket with $\gamma_t = 18.94$,*

$$\text{Bucket Area in eV-s} \cong 0.344 \sqrt{\text{V in MV}} \; \sqrt{\text{p in GeV/c}}$$

$$\text{Bucket Half Height } (\Delta p/p) \cong 0.0144 \sqrt{\text{V in MV}} \,/\, \sqrt{\text{p in GeV/c}}$$

These are shown in Fig. 1 for p = 150, 500, and 1,000 GeV/c as a function of V. Because of the inductive wall impedance,[18] there will be a beam-induced voltage and the effective voltage V^* will be always less than the applied RF voltage V. One can evaluate this voltage reduction in a consistent manner for the elliptic charge distribution.[8] Fortunately, the effect is not expected to be significant in the doubler at 150 GeV/c unless $|Z/n|$ is much larger than $\sim 30\Omega$:

150 GeV/c, 10^{11}/bunch

V		$\lvert Z/n\rvert$ =	10Ω	20Ω	30Ω
0.5 MV		V^*/V =	0.969	0.940	0.912
2. MV		=	0.979	0.959	0.940

* Tom Collins recommends $\nu = 19.6$ (instead of the customary 19.4) for the $\bar{p}p$ colliding mode for which the transition γ is 18.94.

The effect of the wall impedance may be nontrivial at 1 TeV/c, see Fig. 1A, especially for the beam emittance less than $\sim$ 2 eV-s. Once the bucket is specified, the matched bunch length $\pm\phi_B$ and the matched momentum spread $\pm(\Delta p/p)_B$ of the bunch depend only on the bunch emittance A:

$$\phi_B^{\,2}\ (\text{in rad}^2) \simeq 9.6\ \left(1 - \sqrt{1 - (10R/3\pi)}\ \right)$$

where $R \equiv A/(\text{bucket area})$,

$$(\Delta p/p)_B = \sin(\phi_B/2)\cdot(\Delta p/p)_{\text{bucket}}$$

The expression for ϕ_B is not valid when ϕ_B is very large* but the expression for $(\Delta p/p)_B$ is exact once ϕ_B is specified. The bunch length ϕ_B and the momentum spread $(\Delta p/p)_B$ are shown in Figs. 2 and 3, respectively, at p = 150, 500 and 1,000 GeV/c. In using these figures at 1 TeV/c, it is probably more accurate to use V^* (found from Fig. 1A for a given $|Z/n|$) instead of V itself.

microwave instability

A detailed discussion on the threshold criterion for the microwave instability is given in ref. 8. It is shown there that the threshold value of the effective coupling impedance $|Z/n|_{MW}$ is rather insensitive to the ratio $|Z/n|_{MW}/|Z/n|$ where $|Z/n|$ is the low-frequency wall impedance (inductive) as long as this ratio is not much less than unity. The threshold value of $|Z/n|_{MW}$ is plotted in Fig. 4 as a function of the bunch area for p = 1 TeV/c, 10^{11}/bunch, V = (0.5 - 2)MV and $|Z/n|_{MW} = |Z/n|$. The stability condition given by Gareyte is[6]

$$T_m^3 > 5.8\times10^{-4}\ \frac{N(\text{in } 10^{10})}{V(\text{in MV})}\ \frac{R}{F(\text{in MHz})}\ |Z/n|_{MW}$$

where T_m is the full bunch length in meters, N is the number of

* The amount of overestimate is less than 1° for $\phi_B < 120^\circ$.

particles in each bunch, V is the peak RF voltage, F is the RF frequency and R is the average machine radius in meters. Although his model is different in many respects from the one used in ref. 8, there is no essential difference in the result.* Examples obtained from this criterion are also shown in Fig. 4. For the same values of beam area and RF voltage, the bunch length is shorter for higher beam momenta (see Fig. 2) and the threshold value of $|Z/n|_{MW}$ decreases very rapidly (cubic power of the bunch length) with the beam momentum. Nevertheless, even at $p = 1$ TeV/c, the threshold value is so high that the microwave instability is quite unlikely. For example, with $|Z/n|_{MW} = 30\Omega$, the bunch area must be less than 1.5 eV-s in order for the instability to develop. At present, there is no reason to suspect that the coupling impedance might be larger than 30Ω or the beam area would be less than 2 eV-s.

<u>coherent modes instability</u>

Ever since the work by Ruggiero and Vaccaro[19] and one by Keil and Schnell[20] on the longitudinal coasting beam stability based on the dispersion relation, it is customary to express the stability criterion as an approximate condition under which the Landau damping is lost. If S is the spread within the bunch of the angular frequency of synchrotron oscillations and $\Delta\omega_m$ is the coherent angular frequency shift for mode m arising from the coupling impedance of the entire ring, the condition takes the form

$$S > k_m \; |\Delta\omega_m|$$

with a mode constant k_m which, according to Sacherer,[11] is approximately $4/\sqrt{m}$. For perfectly conducting walls and a bunch with a parabolic line density, $\Delta\omega_m$ is real and proportional to $\sqrt{m}$. The necessary spread S is then independent of the mode number m. The behavior of $\Delta\omega_m$ as a function of m is of course different for resistive walls and resonator-like objects. Without any special cavity such as

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* The expression is valid for the bunch size much shorter than the bucket size. For larger bunch size, the expression should still be good for scaling purposes.

high-harmonic Landau cavities, the lowest-order expression for S as a function of the bunch length ϕ_B is*

$$S = (\phi_B{}^2/16)\cdot\omega_s \qquad (\omega_s = \text{synchr. freq.})$$

A more rigorous treatment of the stability in the case of a pure inductive impedance and a parabolic bunch has been given recently by Besnier. I have not seen Besnier's report and the following description of his result is taken from refs. 2 and 6. The stability condition by Besnier can be written in the form

$$(S/\omega_s) > k_m|Z/n|\cdot 1.42\times10^{-3}\ \frac{N(\text{in } 10^{10})\ h\ F(\text{in MHz})}{V(\text{in MV})\ \phi_B{}^3(\text{in degrees})}$$

where h is the harmonic number, F is the RF frequency and N is the number of particles in a bunch. Coefficients k_m are:

	mode	k_m
m = 1	rigid dipole mode	3.4
= 1	non-rigid dipole mode	1.4
= 2	quadrupole mode	1.6
= 3	sextupole mode	0.9
= 4	octupole mode	0.65

If the lowest-order expression for S is used, one finds, with h=1113, N=10 and F=53.1,

$$k_m|Z/n| < 0.227\times10^{-7}\ V^*(\text{in MV})\ \phi_B{}^5(\text{in degrees})$$

where V^* is the reduced voltage. The relation is plotted in Fig. 5.

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* The exact expression for the oscillation period is, for a stationary bucket,

$$T(\phi_B) = (2/\pi)K(\phi_B)T(0)$$

where $K(\phi_B)$ is the complete elliptic integral of the first kind,

$$K(\phi_B) = \int_0^{\pi/2} d\theta\ \ (1 - m\sin^2\theta)^{-1/2}\ \ ;\ \ m = \sin^2(\phi_B/2)$$

To use this criterion, one should proceed as follows:

1. Fix the voltage V, the beam area and $|Z/n|$ (example: V=1.5 MV, beam area=2 eV-s, $|Z/n|=20\Omega$).
2. Find V^*/V from Fig. 1A (if possible) and calculate V^* (at 1 TeV/c, $V^*/V=0.842$, $V^*=1.26$ MV).
3. Find ϕ_B from Fig. 2 using V^* instead of V (beam area/$\sqrt{V^*}$ = 2/$\sqrt{1.26}$ = 1.78, $\phi_B=53.5^o$ at 1 TeV/c).
4. Use the above criterion to find

$$k_m\ 20. < 0.227\times10^{-7}\times1.26\times(53.5)^5 = 12.5$$

or $k_m < 0.6$ for stability.

For this example, all modes up to $m = 4$ can be unstable. It should be remembered here that the criterion is only for the loss of the Landau damping. The growth rate of amplitude is decided by other mechanism so that the instability could be relatively fast or very slow. For $|Z/n| < 20\Omega$, one can probably ignore the difference between V and V^* in the above criterion. With the reasonable assumption $|Z/n| = (20\sim30)\Omega$ and beam area = $(2\sim2.5)$eV-s, it seems difficult to avoid the coherent oscillations of mode 1 (dipole) and 2 (quadrupole). This can be seen from the following table for which $V^* = 0.5$ MV (V = 0.5∿0.7 MV depending on the beam area and $|Z/n|$) has been assumed.

	$\|Z/n\|=20\Omega$ min. ϕ_B	$\|Z/n\|=20\Omega$ min. area	$\|Z/n\|=30\Omega$ min. ϕ_B	$\|Z/n\|=30\Omega$ min. area
m=1	90^o	3.25 eV-s	98^o	3.76 eV-s
=1*	75.5	2.39	82	2.77
=2	77.5	2.50	84	2.88
=3	69	2.03	75	2.36
=4	65	1.81	70	2.08

* non-rigid dipole motion.

Beam storage studies in the SPS with the intensity of 10^{11} per bunch have demonstrated the existence of dipole, quadrupole and sextupole mode oscillations. With the damping of the rigid dipole mode by the phase loop and of the quadrupole mode by an amplitude feedback system, they succeeded in increasing the beam lifetime from two hours to four hours. One of the most significant findings in their beam studies is the phenomenon of overshooting.[21] If the initial bunch size is less than the minimum value given by the stability criterion, it eventually grows to a value larger than the threshold value. Furthermore, the smaller the initial value, the larger the final amplitude so that the optimum bunch length is the threshold value specified by the stability criterion. So far, the overshoot relation is established only qualitatively and it is difficult to predict the magnitude of the final amplitude for a given initial bunch length.

IV.

If one compared the CERN interpretation with the report written by Ruggiero last year (ref. 5, UPC No. 72), one would notice the difference in terminology. Ruggiero uses the word "microwave instability" to describe all longitudinal oscillations of a single bunch regardless of their growth time whereas CERN people treat them as coherent mode oscillations studied by Sacherer.[11] Since there is no single bunch mode with a growing oscillation amplitude, one must find some mechanism to induce an actual growth and this naturally gives a rather long growth time. In spite of this (superficial) difference, I believe a set of recommendations given by Ruggiero are still valid. The following comments are simply my random thoughts at this time and should be treated as such.

1. "Bunch Spreader" I do not feel as strongly as Sandro on the necessity of this device. Surely it would be nice to be able to control the bunch emittance precisely. However, my instinct tells me that longitudinal "super" dampers would be easier to build and to

operate than a precision bunch spreader. For a dozen or so bunches in the ring, a bandwidth of a few MHz is sufficient.

2. Higher harmonic cavity (Landau cavity). I have a high hope for the usefulness of this. It is certainly very effective in creating a large spread in the synchrotron oscillation frequency within a bunch and that is what we need to suppress the coherent mode instabilities. I am not convinced that, for our case, the effect of RF noise would become much worse with a larger spread. The bunch length relative to the bucket size is much shorter in our case compared to the SPS. The larger spread will also be useful in controling any residual coupled-bunch instabilities. This type of cavity is the only device to suppress the instability if it is caused by a very short-range field which induces an energy loss (Ruggiero model[9]).

3. Measurements and calculations by Bob Shafer of the coupling impedance of various devices (pickups, kickers, Lambertsons, bellows, etc.) are extremely important. If the total $|Z/n|$ can be reduced to less than ten ohms, we may not see any coherent mode oscillations.

4. Finally, it is important to follow the doctrine of "First thing first". Try to understand the effect of noise (RF or otherwise) on a stored bunch. Be sure that the RF cavities do not induce strong coupled-bunch instabilities.

REFERENCES

1. S. Ohnuma, UPC No. 118 (12/20/79); UPC No. 118 Addendum(12/27/79).
2. SPS Improvement Reports No. 154 (22nd January, 1979) and No. 162 (19th July, 1979).
3. EXP-83 (8/15/77), EXP-84 (8/25/77), EXP-84A (9/16/77), EXP-96 (3/16/79).
4. I understand Jim MacLachlan is preparing a report and it will be available soon.
5. A. G. Ruggiero, UPC No. 70 (1/12/79); UPC No. 71 (1/17/79); UPC No. 72 (1/8/79); UPC No. 81 (January 1979).
6. J. Gareyte, CERN/SPS/80-1(DI), February, 1980.
7. D. Boussard, LABII/RF/Int./75-2, April 24, 1975. This is a very interesting but hard-to-find report.
8. For example, see TM-749 (10/24/77), section III.
9. A. G. Ruggiero, Xth International Conference on High Energy Accelerators, Protvino, July 1977, vol II., p. 260.
10. D. Boussard, LABII/RF/Int. Note/75-8. Ihave learnt from Boussard that EXP-74 is not quite right and $|Z/n|$ for the microwave instability in the main ring should be 20 ohms or less (private communication, April 21, 1978). The report quoted here was sent to us by Boussard in 1975 but it was somehow never made available to the general public (an intentional sabotage?). Boussard kindly sent me a copy in 1978 when I exchanged letters with him regarding the possible microwave instability in the main ring. I record this here for a possible historical interest.
11. F. Sacherer, IEEE Trans. Nucl. Sci., NS-20, No. 3, p. 825 ('73).
12. F. Sacherer, IEEE Trans. Nucl. Sci., NS-24, No. 3, p. 1393('77).
13. SPS $p\bar{p}$ Study Team, CERN/SPS/AC/79-18.
14. S. Hansen, et al., IEEE Trans. Nucl. Sci., NS-24, No. 3, p.1452 ('77).
15. A Report on the Design of the Fermi National Accelerator Laboratory Superconducting Accelerator, May 1979. p.173.
16. ibid., p. 169.
17. ibid., p. 153.
18. S. Hansen, et al., IEEE Trans. Nucl. Sci., NS-22, No. 3, 1381('75).
19. A. G. Ruggiero and V. G. Vaccaro, CERN-ISR-TH/68-33 (1968).
20. E. Keil and W. Schnell, CERN-ISR-TH-RF/69-48 (1969).
21. R. A. Dory, MURA Report No. 654 (1962).

Fig. 1. Stationary Bucket: area and half-height

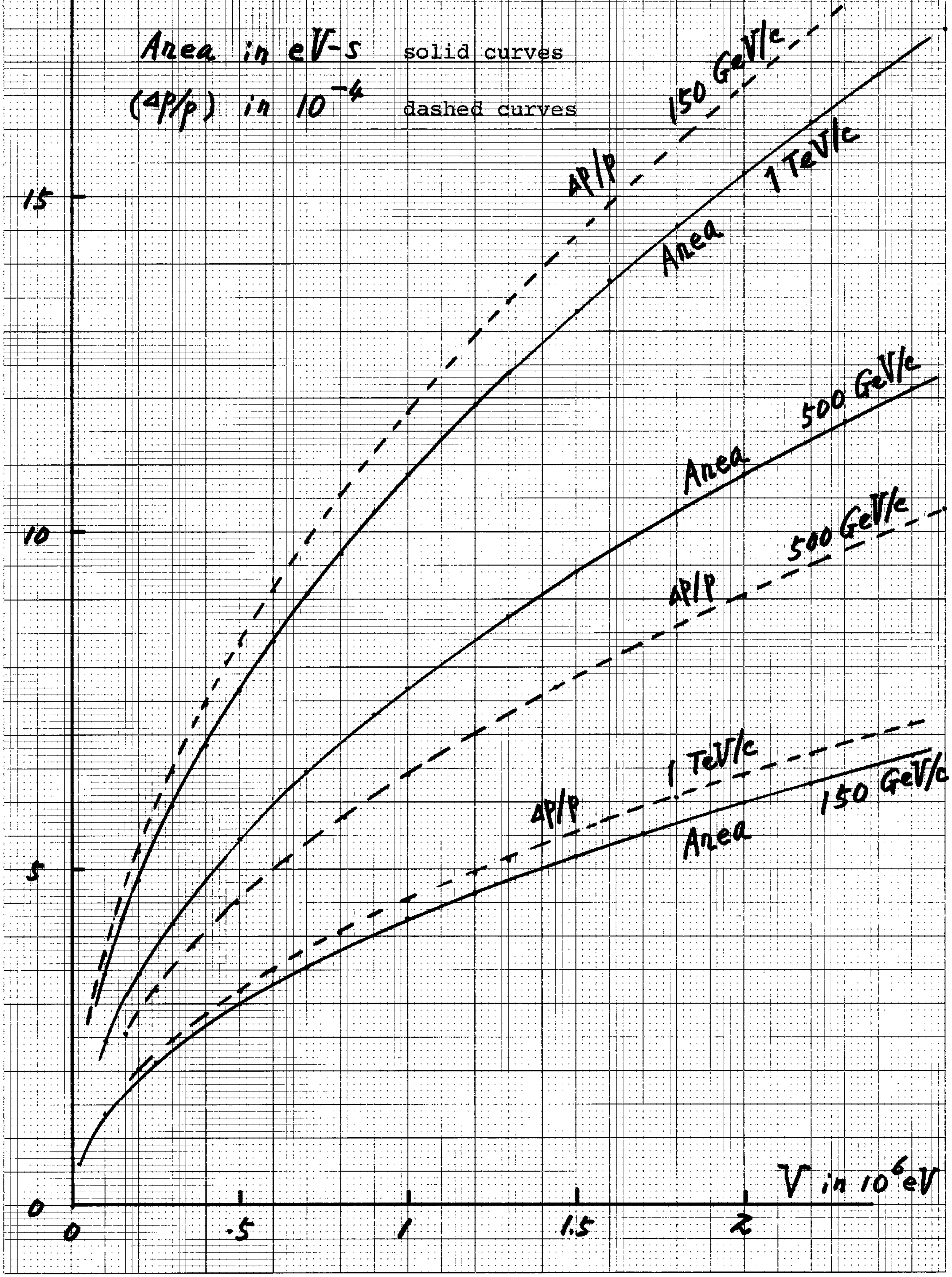
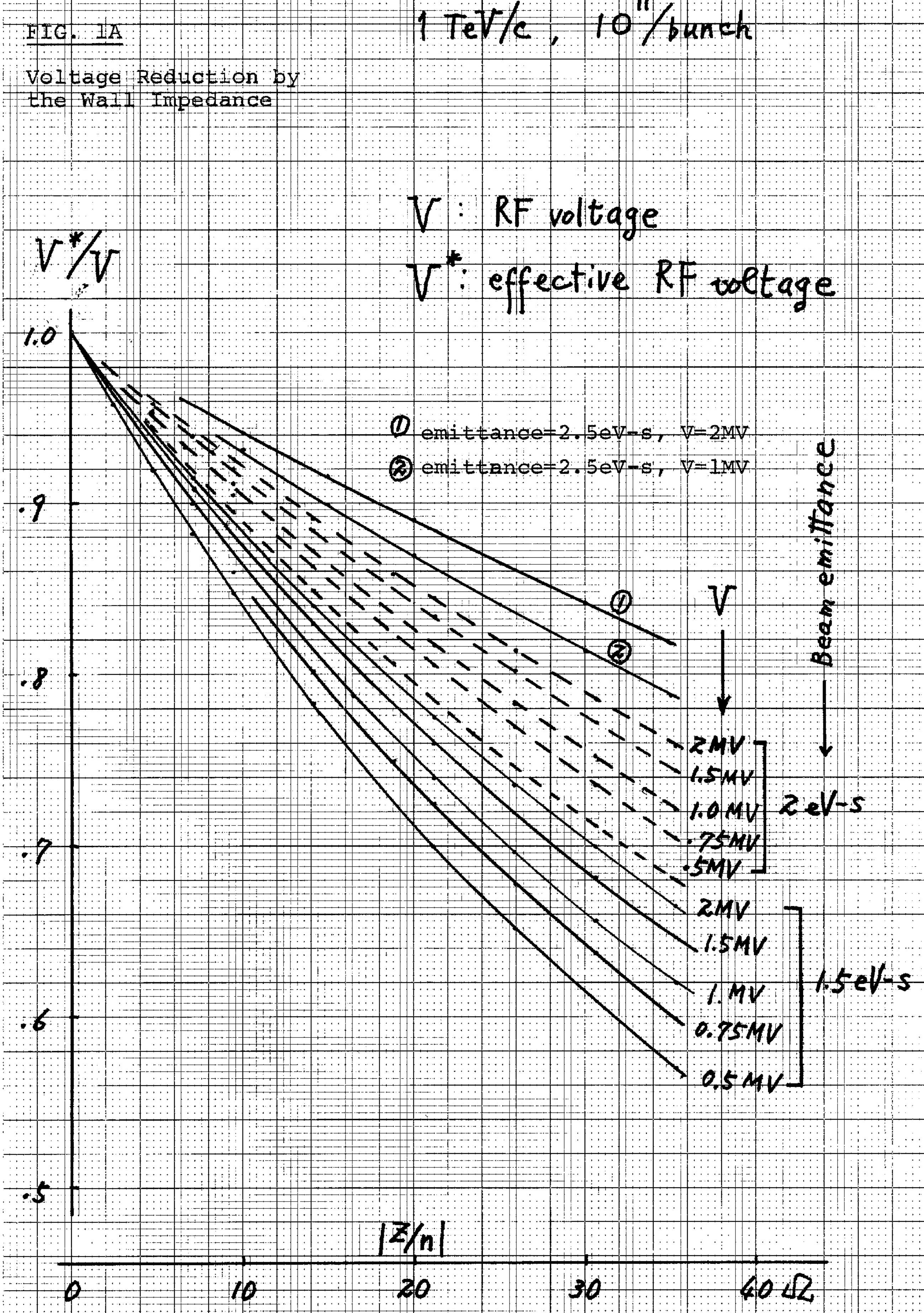


FIG. 1A

Voltage Reduction by the Wall Impedance



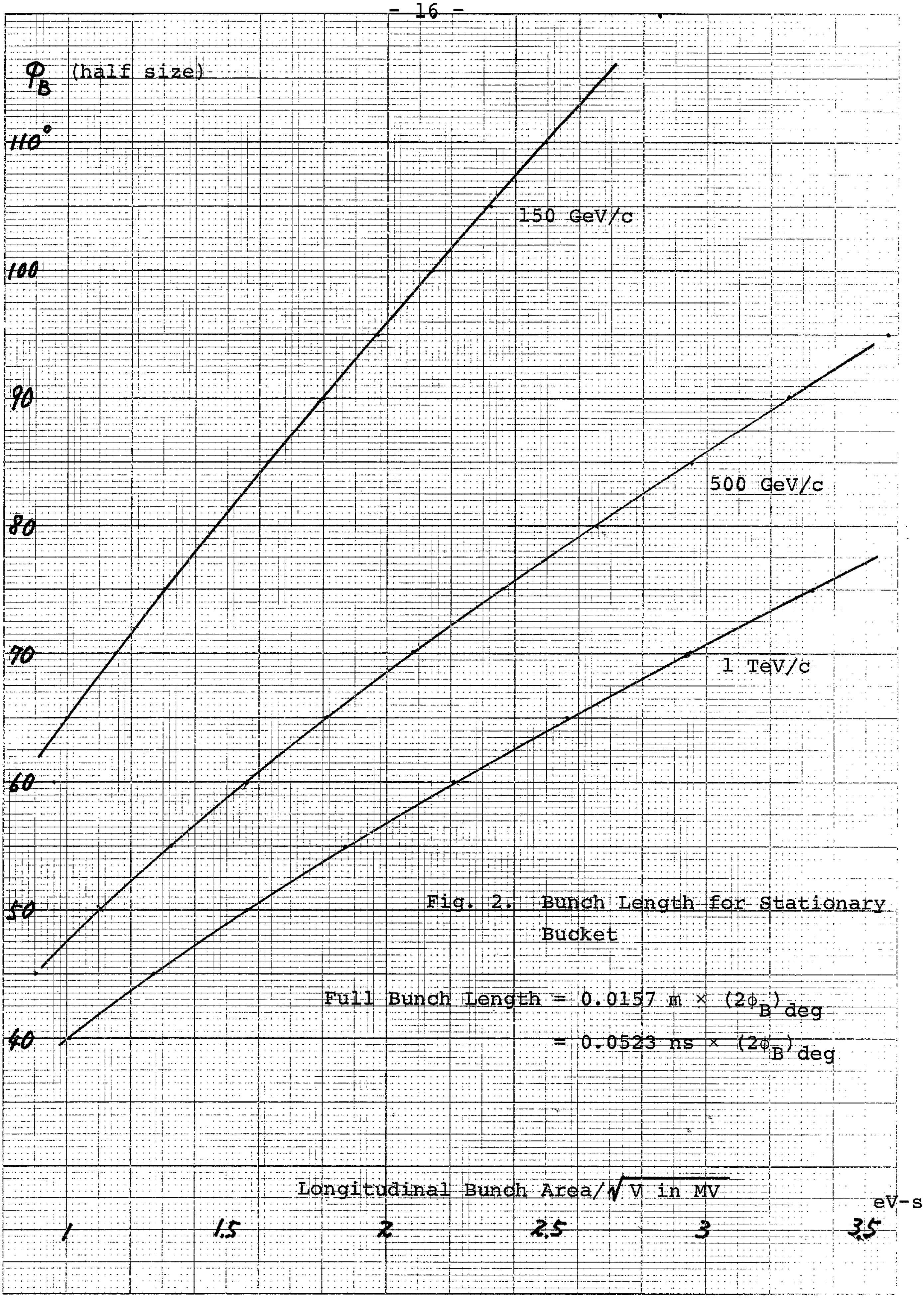


Fig. 2. Bunch Length for Stationary Bucket

Full Bunch Length = 0.0157 m × $(2\phi_B)_{deg}$

= 0.0523 ns × $(2\phi_B)_{deg}$

Fig. 3 Bunch Momentum Spread (Half Height)

STATIONARY BUCKET

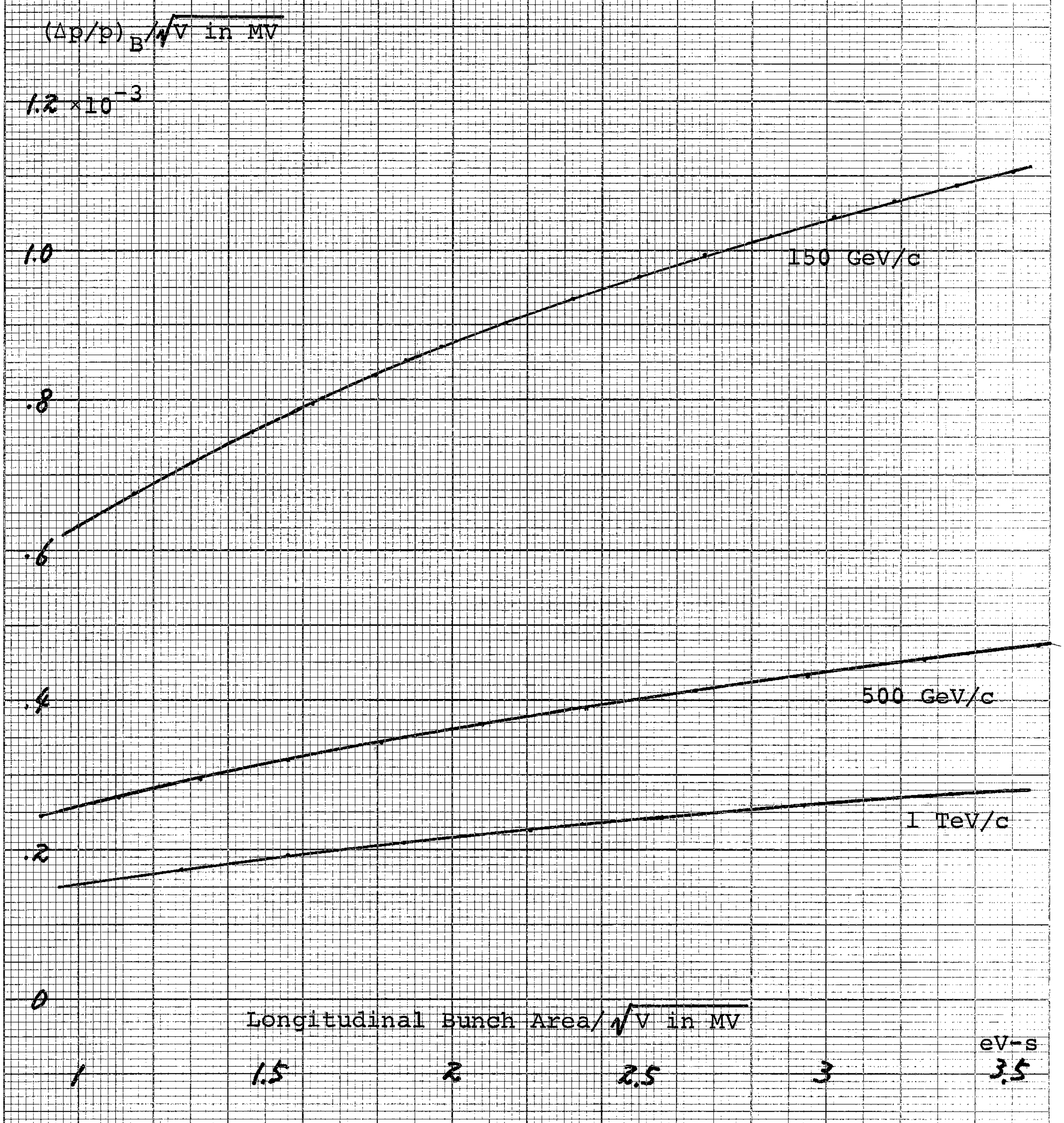
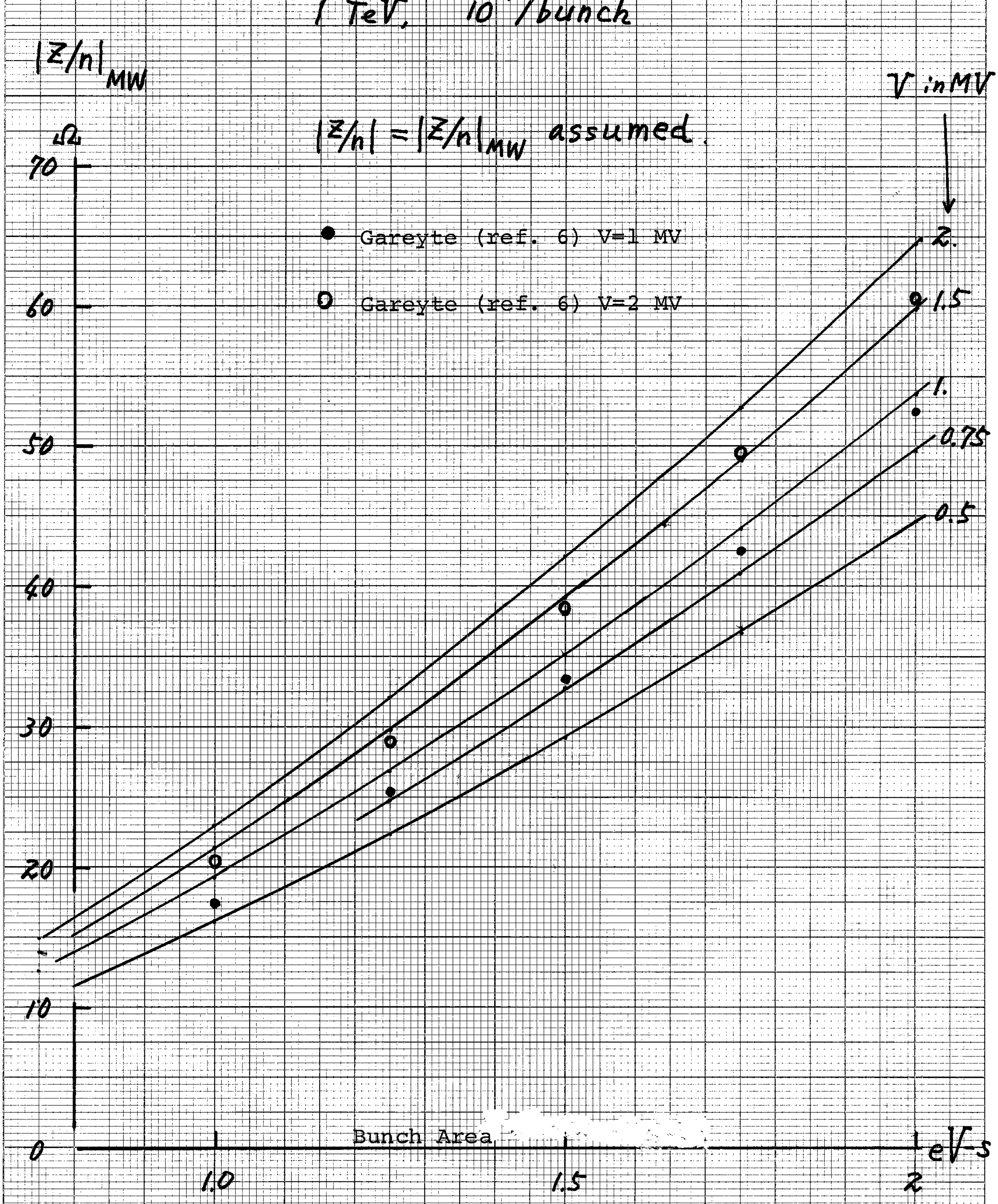


Fig. 4 Threshold of the Microwave Instability



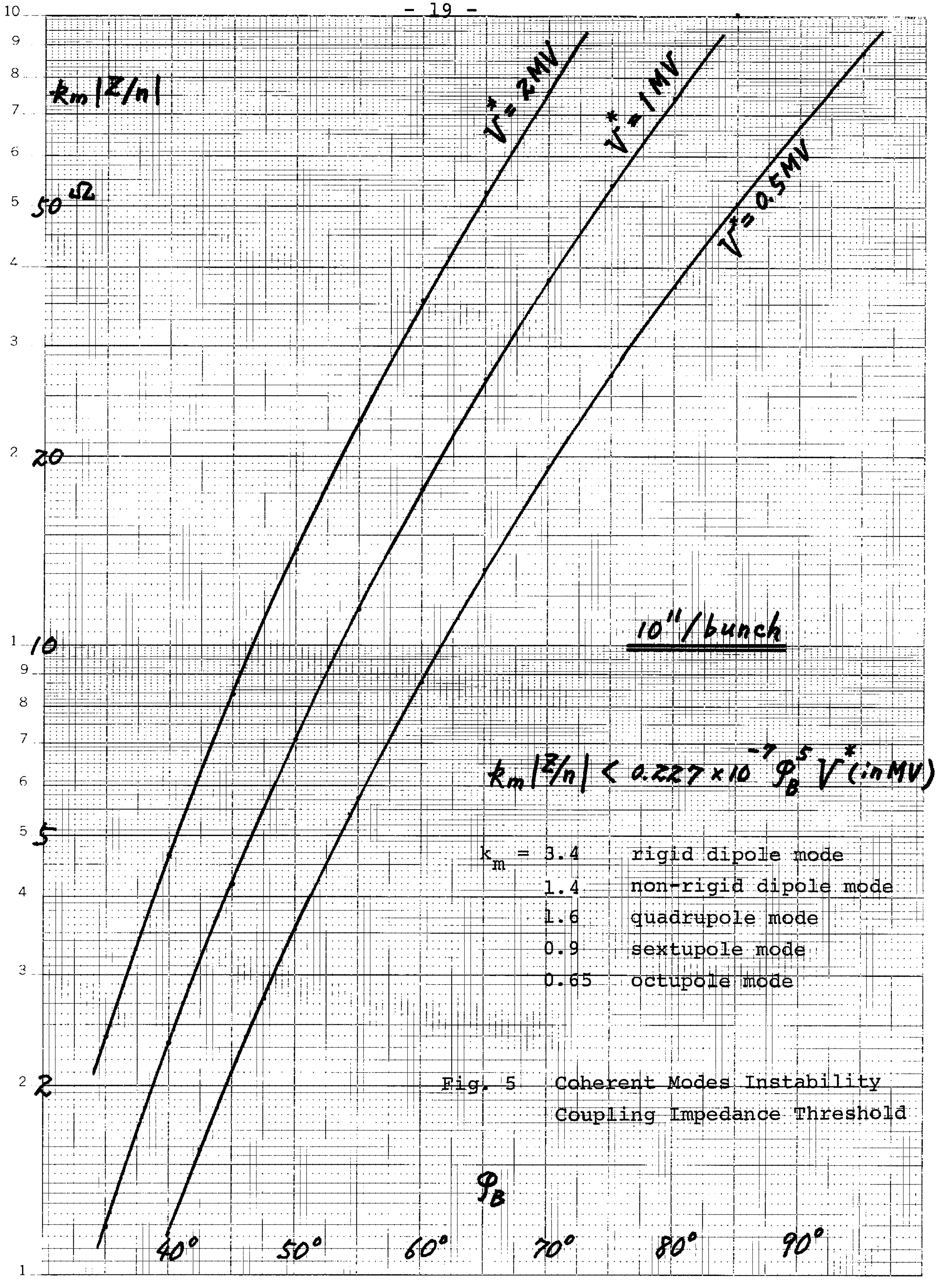


Fig. 5 Coherent Modes Instability Coupling Impedance Threshold

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Addendum to "LONGITUDINAL INSTABILITIES REVISITED"

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J. Gareyte of the CERN SPS Division kindly sent me a letter answering my questions regarding the current CERN interpretation of the longitudinal instabilities of a single bunch.

1. People working on the LEP project approximate the broad-band impedance by a resonator with Q = 1 and the resonant frequency in the GHz range, vacuum pipe cut-off. For this case, Im(Z/n) at low frequencies is equal to Re(Z/n) at high frequency. This means that, in UPC No. 128, $|Z/n|_{MW}$ which is the magnitude of the impedance at high frequencies (many wavelength within the bunch) is equal to the magnitude of Im(Z/n) at low frequencies.

2. The voltage reduction is caused by Im(Z/n), the inductive part of the impedance.

3. For very high mode numbers and for very short bunches, one cannot use Im(Z/n) at low frequencies to determine the coherent modes instability thresholds.

4. I learnt about the convenience of using the elliptic charge distribution (see UPC No. 128, p. 6) from Frank Sacherer when we worked together during the summer of 1977 at Brookhaven. Gareyte imformed me of a recent article by Hofmann and Pedersen, IEEE Trans. Nucl. Sci., NS-26 (1979), 3526, which is quite educational. That was the last of many things I learnt from Frank and the elliptic distribution is therefore very special to me.