FERMILAB-LU-004

RFQ Linac and its limit and Problem

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Outline

- Introduction
- Para-axial field
- Beam Dynamics
- RF structure
- Limit and problem

1. Introduction

In the 50's the rf quadrupoles had been used for focusing of charged particle beam and rf massspectrography.

(J. P. Blewett, Phys. Rev. 88, 1197 (1952))

(W. Paul et. al., Z. Physik 140 (1955)

In their cases only the two dimentional field had been used (no longitudinal variation)

that is only the following field

$$u(r,\varphi) = \frac{1}{2} V \sum_{s=0}^{\infty} A_{0s} r^{2(2s+1)} \cos 2(2s+1)\varphi \qquad (1.1)$$

had been used.

The contribution of I. M. Kapchinskij and V. A. Teplyakov is that they introduced three dimentional field

$$u(r,\varphi,z) = -\frac{V}{2}\Big[\sum_{s=0}^{\infty} A_{0s} r^{2(2s+1)} \cos 2(2s+1)\varphi +$$

$$\sum_{n=1}^{\infty}\sum_{s=0}^{\infty} A_{ns} I_{2s}(knr)\cos 2s\varphi \sin knz\Big] \qquad (1.2)$$

The first term in (1.2) is pure two-dimensional multipole field. The second one in (1.2) gives longitudinal field compoment E_z.

The contribution of Los-Alamos group is that they simplified the field expresion of (1.2) and only took two lowest order term:

$$U_0(r,\varphi,z) = -\frac{V}{2}\left[A_{00}\, r^2 \cos 2\varphi + A_{10} I_0(kr) \sin kz\right] \quad (1.3)$$

Using (1.3) if the shape of vane is a perfect hipebol cross-section you will have the precise field distribution determined by (1.3).

And the Los Alamos group built first prototype of RFQ Linac with high transmision efficiency (> 90%), and they wrote a series of computer programms for beam dynamics design.

Soon after that it had found that the tolerance of the intervane voltage is very tight

$$\frac{\delta V}{V} = -\frac{1}{6}\frac{L^2}{\lambda\!\!\!^{-2}}\frac{\delta c}{c} \quad (1.4)$$

$$\frac{\delta c}{c} \approx -\frac{\delta g}{g} \quad (1.5)$$

where V — intervane voltage
δV — deviation of V
L — the total length of the RFQ Linac
$\lambda\!\!\!^{-}$ — rf wave devided by 2π

C — capacitance between adjacent vanes

g — the average intervane gap

δC — the deviation of C

δg — the deviation of g

Example: if you want to keep $\frac{\delta V}{V} \lesssim 10\%$

($L \approx 3\,m$, $f = 425\,MHz$) the $\delta g \lesssim 0.003\,mm$.

(D. D. Armstrong. et. al. RFQ Devel. At Los Alamos)

Field stabilization is important.

First at Los Alamos used manifold (J.M. Potter 1979 Linear Acc. Conf.)

Then several RFQ structures had been proposed, but at present two of them are more successful than orthers. They are

- vane coupling rings (VCR)

(Proposed by H. Lancaster et. al. 12th Int. Acc. Conf., FNAL, P. 512 (1983))

- Resonant Loop Coupling (RLC)
(Proposed by A. Schempp. 1986 Linear Acc. Conf. P. 251, SLAC (1986))

2. Para-axial field

If you look around the axis in RFQ cavity the field near axis can be discribed by Laplace equation (quasisteady state)

$$\frac{\partial}{\partial r}\left(r\frac{\partial \phi}{\partial r}\right)+\frac{1}{r}\frac{\partial^2 \phi}{\partial \vartheta^2}+r\frac{\partial^2 \phi}{\partial z^2}=0 \qquad (2.1)$$

The general solution is

$$\Phi = e^{\pm pkz}\begin{matrix}\cos n\vartheta \\ \sin n\vartheta\end{matrix}\left[C_1 J_n(pkr)+C_2 N_n(pkr)\right] \qquad (2.2)$$

$C_2 = 0 \quad \because \; r \to 0, \; N_n(0) \to -\infty$

if we have a sine form modulation along z axis pk should be imaginary $pk \to jpk$ So Bessel

function $J_n(pkr)$ is changed into modified

$$J_n(jpkr) = j^p I_n(pkr) \qquad (2.3)$$

$$\phi = \sin pkz \begin{matrix}\cos n\vartheta \\ \sin n\vartheta\end{matrix} c_1 I_n(pkr) \qquad (2.4)$$

And if chose the symmetric plane of multipole as the initial reference plane then (2.4) can be written as

$$\phi = c_1 \sin pkz \cos n\vartheta \, I_n(pkr) \qquad (2.5)$$

For the property of symmetry of the multipole (including quadrupole) n should be even

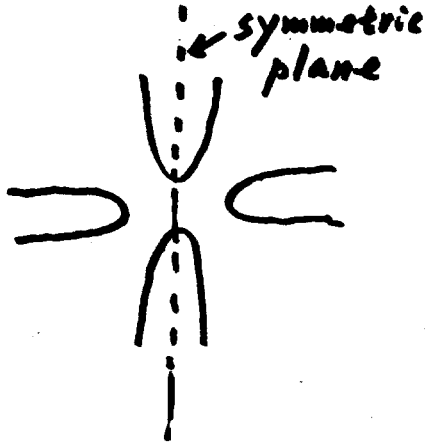
$$\phi_1 = c_1 \sin pkz \cos 2s\vartheta \, I_{2s}(pkr) \qquad (2.6)$$

If the $k=0$ the field is two-dimensional

$$\frac{\partial^2\phi}{\partial r^2} + \frac{1}{r}\frac{\partial\phi}{\partial r} + \frac{1}{r^2}\frac{\partial^2\phi}{\partial\vartheta^2} = 0 \qquad (2.7)$$

Because the quadrupole has to be included

and we have taken symmetrical plane as initial reference plane the solution for (2.7) is



$$\phi_2(r,\vartheta)=\sum_{s=0}^{\infty} A_{0s}\, r^{2(2s+1)} \cos 2(2s+1)\vartheta \qquad (2.8)$$

The general form of (2.6) can be written as

$$\phi_1=\sum_{p=1}^{\infty}\sum_{s=0}^{\infty} A_{ps} I_{2s}(pkr)\cos 2s\vartheta \sin pkz \qquad (2.9)$$

Superposition of ϕ_1 and ϕ_2 is

$$\phi=\sum_{s=0}^{\infty} A_{0s}\, r^{2(2s+1)} \cos 2(2s+1)\vartheta+\sum_{p=1}^{\infty}\sum_{s=0}^{\infty} A_{ps} I_{2s}(pkr)\cos 2s\vartheta \cdot \sin pkz \qquad (2.10)$$

According to (2.10) we have following field components:

A_{10} — axial symmetric field

A_{00}, A_{11} — quadrupole

A_{12} — (8 pole) Octapole

A_{13}, A_{01} — (12-pole) Duosixtopole

A_{14} — (16-pole) ? (how to call in english)

A_{02} — (20-pole) Duodesimal

⋮

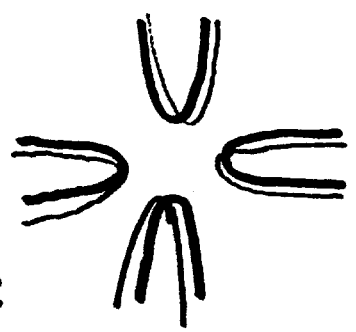
This is the ideal case in which the four electrods (whatever geometric shape they have) are exact symmetric around the axis. In practice the four electrods can not be assembled so precisely. So in general case the dipole, sixtapole, 10-pole, 14-pole ... are also exist

$$A_n \left(\frac{r}{r_0}\right)^n \cos n\vartheta \qquad (2.11)$$

$$n = 0, 1, 2, 3, \cdots$$

the blue one is ideal case

the red one is non-ideal case

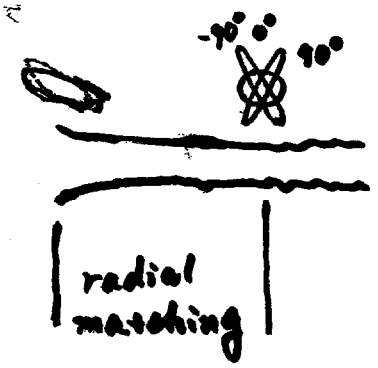


3. Beam dynamics

Two points have to be noticed

① From time independent acceptance to time dependent (rf) acceptance.

How to match ?



First chose radius of aperture at the end of radial matching section and give the emmitance from the ion source to determine the shape of acceptance ellipses relative to different rf field phases (-90°, 0°, 90°, 180°...). Then trace these ellipses back to the input (5-10 particles)

Because the strength of field $B = \frac{e\lambda^2}{m_0 c^2} \frac{XV}{a^2}$

is decreasing toward the input. So the three ellipses at the input have small difference.

② The transverse and longitudinal current limits are determined at the end of gental buncher. This is because at the end of gental buncher $\varphi_s = -35° - -30°$ that is almost the final synchron phase. The angular length of the separatrix Φ is determined by

$$\tan \varphi_s = \frac{\sin \Phi - \Phi}{1 - \cos \Phi} \qquad (3.1)$$

and the bunch space length Z_b is:

$$Z_b = \frac{\beta \lambda \Phi}{2\pi} \qquad (3.2)$$

From (3.1) and (3.2) it is obvious that at the end of gental buncher

is the minimum (which is kept constant through the acceleration section) and the space charge density is the maximum.

According to the formulas (smooth approximation T. P. Wangler)

$$I_t = \frac{4\mu_t m_0 c^2 \beta \gamma^3 |\phi_s| \sigma_0^2}{3 Z_0 e q N^2 \psi [1-f(p)]} \left(\frac{a}{\lambda}\right)^2 \qquad (3.3)$$

where μ_t — ratio of transverse space charge force to focusing force

σ_0 — phase advance at zero current

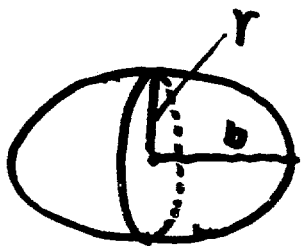
N — ratio of number of focusing period per $\beta\lambda$

$f(p)$ — space charge form factor

$$I_l = \frac{8\pi^2}{3 Z_0} \mu_l \frac{r^2 b}{f(p)} \frac{1}{\beta\lambda^2} E_0 T |\sin\phi_s| \qquad (3.4)$$

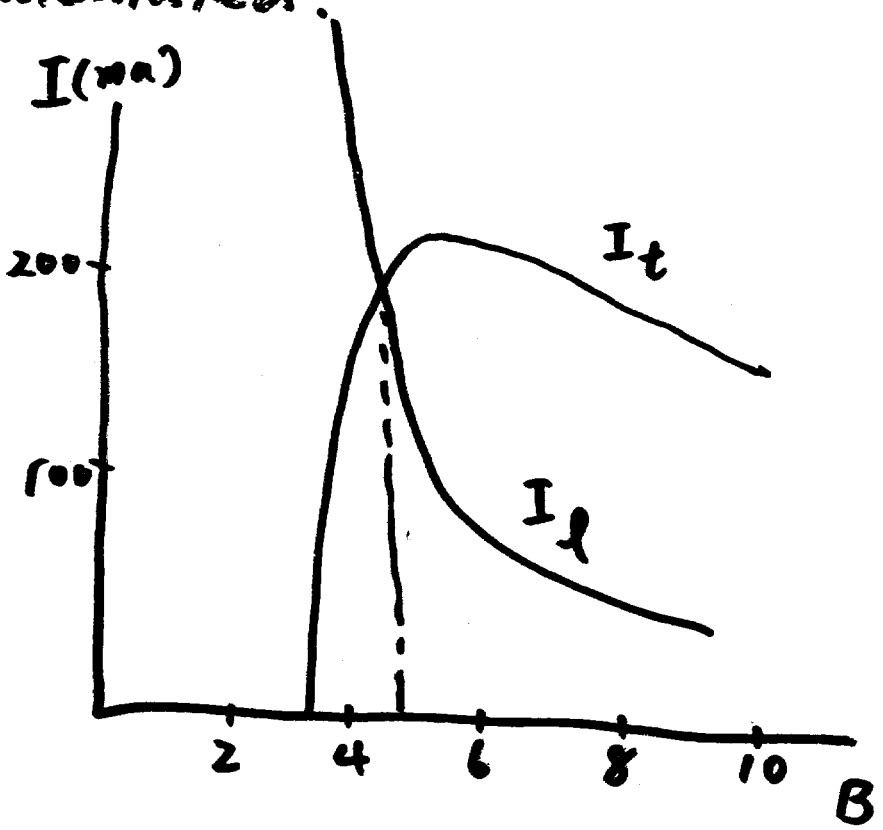
where μ_l — ratio of longitudinal space charge force to the focusing force

for space charge the ellipsoid model is used



sopose bunch has a ellipsoid shape.

then the I_t (trans. space charge limit current) and I_l (longitudinal sp. ch. limit current) are calculated.



4. RF Structure

According to the frequency RF structure can be devided into two categerys:

a) High frequency

Four vane structure which itself involves many substructures: (See Table 1)

VCR — Vane Coupling Rings (Lancaster, 1983)
RLR — Resonant Loop Rings (Schempp, 1986)
DDR — Double Dipole Resonator (Hutcheon, 1982)
AHS — Alternating Half Short (Hutcheon, 1982)
(B. Wang, 1981)

The above listed structures are for transverse field stabilization.
There are very few works on the longitudinal field stabilization:

RLC — Resonant Line Coupling (Schempp 1986)

3-D VCR — three dimensional vane coupling rings (proposed by B. Wang 1987)

b) Low frequency RFQ structures

Split Coaxial Resonator

Four Rod < straight stems (Frankfurt Uni. and Chalk River)
spiral

Double-H type Resonator (for high frequency) (first by Soviet Union)

Low frequency structure is used for heavy ion acceleration (β is low)

Comparison between 4-rod and 4-vane structure (201.25 MHz)

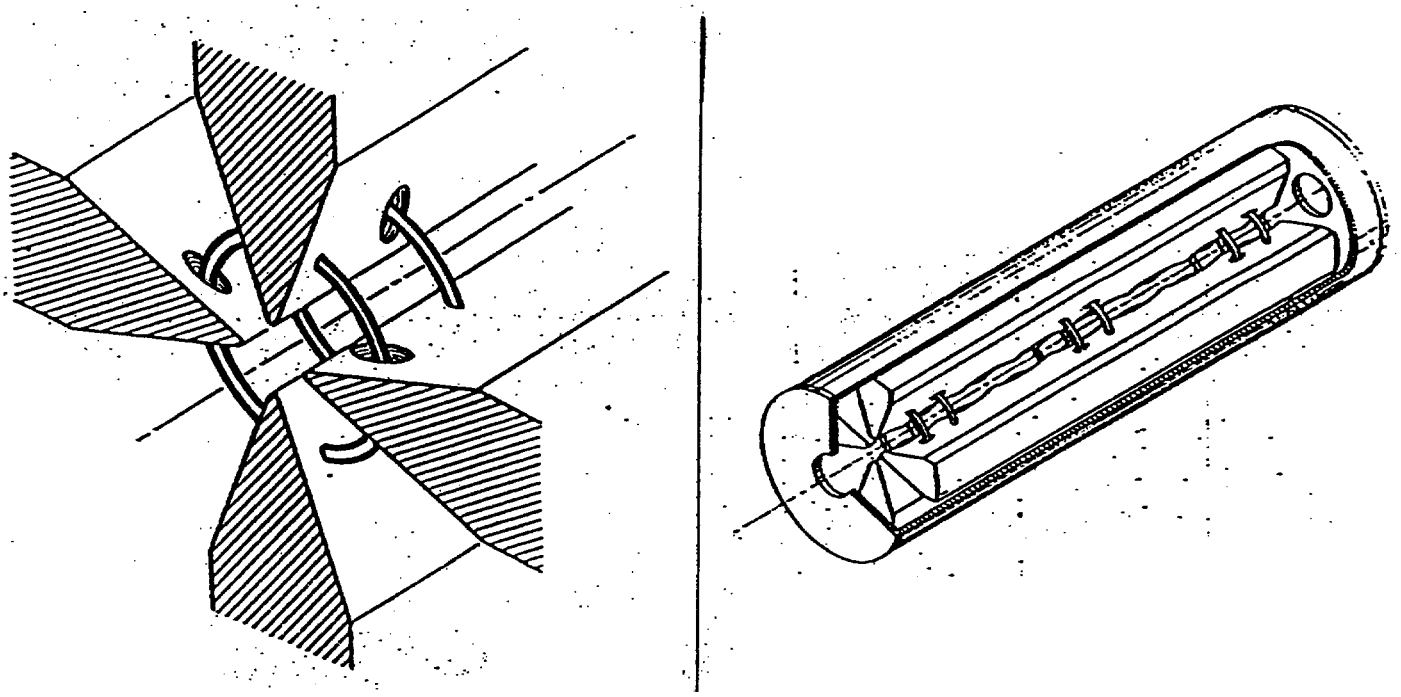
① In principle rf power loss in 4-vane cavity smaller than that in 4-rod stem structure.

② 4-rod use trapezoidal shape to increase accelerating efficiency $\lesssim 10\%$, so the net rf efficiency may be the same as for the four vane structure. But 4-vane can also use trapezoidal shape tip. Then 4-vane structure will have higher rf efficiency than 4-rod structure.

③ 4-rod structure is simpler than 4-vane structure.

* VANE COUPLING RINGS (VCRs): STABILIZES INTERNALLY THE 4-V STRUCTURE, SO THE CAVITY CAN BE LOOP-DRIVEN INSTEAD OF MANIFOLD-DRIVEN. THE VCRs ELIMINATE THE DIPOLE MODES IN THE FREQUENCY RANGE OF INTEREST, PROVIDE ADEQUATE QUADRANT BALANCE AND SYMPLIFY END TUNERS, BUT THE FREQUENCY OF THE FUNDAMENTAL RFQ MODE IS SHIFTED BY A FEW PERCENT TO A LOWER VALUE BY THE VCR's.

VCR



* ALTERNATING HALF SHORT STRUCTURE OR FOLDED DIPOLE (WANG BO-SI DESIGN)

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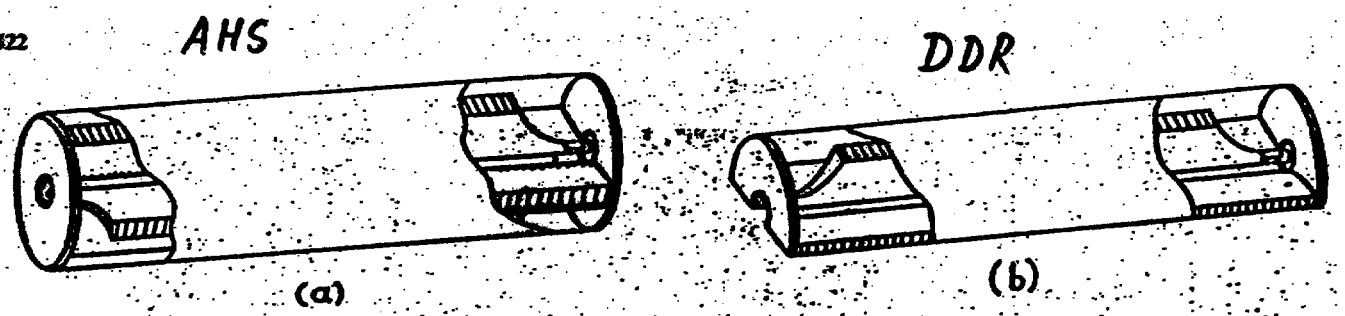
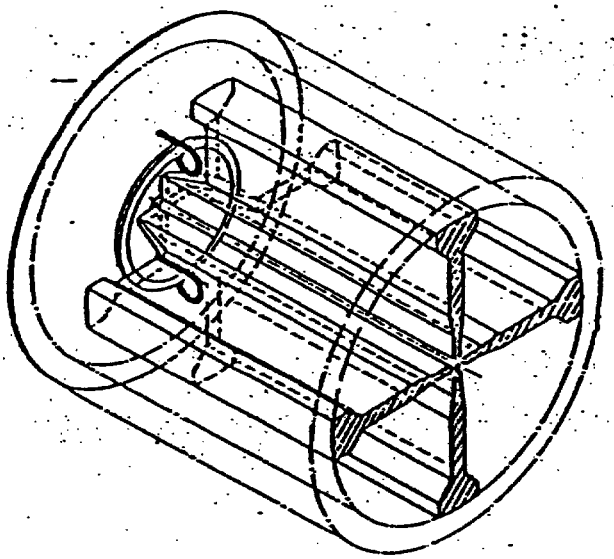


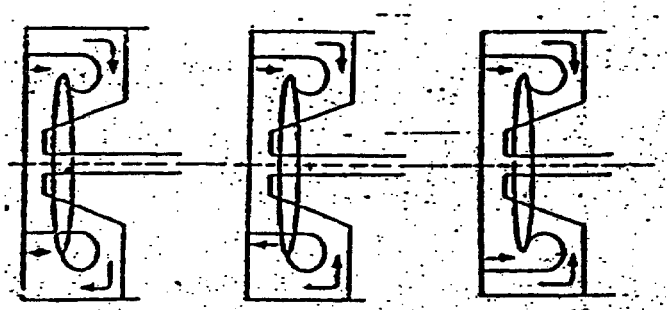
Cutaway diagrams showing two new end termination configurations which increase field stabilities - (a) folded dipole, (b) one half of a double dipole.

* RESONANT LOOP RINGS (RLR)

RLR



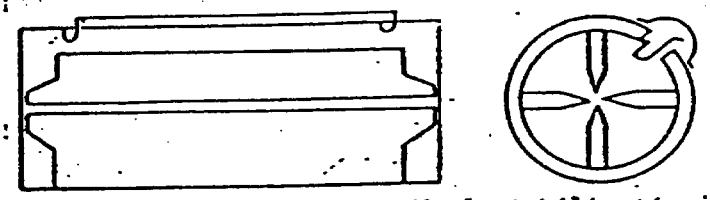
End cell resonant loop ring (RLR) coupler scheme



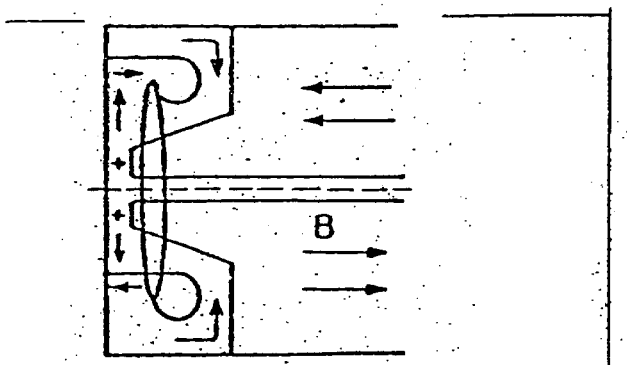
Dipole Quadrupole Quadrupole-Mode

End cell excitation of an RLR

RLC



Set-up for longitudinal stabilisation with one RLC



End cell resonant coupler

Table 1

TABLE OF RFQ LINAC STRUCTURE

STRUCTURES FOR AZIMUTHAL FIELD STABILIZATION

NAME OF STRACTURE	VCR	RLR	DDR	AHS
FEATURE OF STRUCTURE	Shorting opposite vanes by conductor rings	Inserting a conductor ring with two diameterly placed loops into vane end nose	Connecting two resonators of TM_{010}-like mode	Alternatively shorting two pairs of opposite vanes to cavity end walls
OPERATING MODE	disturbed $\frac{\pi}{2}$ mode	disturbed $\frac{\pi}{2}$ mode	π	zero
COUPLING BETWEEN QUADRANTS	Stronger than ordinary RFQ	Stronger than ordinary RFQ	Weaker than ordinary RFQ	approximately two times larger than ordinary RFQ
MODE SEPARATION	larger than ordinary RFQ	larger than ordinary RFQ	larger than ordinary RFQ	larger than ordinary RFQ
GROUP VELOCITY IN AZIMUTH	small	small	zero	zero
AUTHORS	Berkeley group, USA	Frankfurt University West Germany	Chalk River Nucl. Lab. Canada	IHEP, China TAC, USA Chalk River Nucl. L. Canada

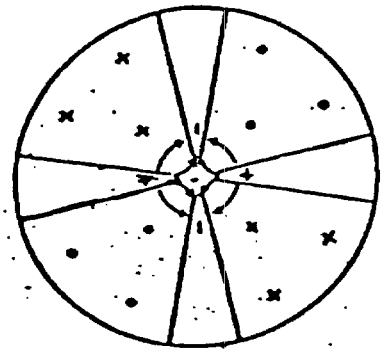
Table 2

STRUCTURE FOR LONGITUDINAL FIELD STABILIZATION

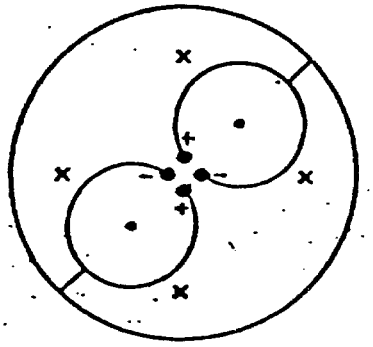
NAME OF STRUCTURE	RLC	3-D VCR*
FEATURE OF STRUCTURE	Similar to RLR	Connecting of opposite vanes by inclined rings
OPERATING MODE	?	$\frac{\pi}{2}$ for both azimuthal and longitudinal
COUPLING IN LONGITUDINAL DIRECTION	larger than without RLC	large in both azimuthal and longitudinal directions
MODE SEPARATION	larger than without RLC	large in both azimuthal and longitudinal
GROUP VELOCITY IN LONGITUDINAL		highest for both azimuthal and longitudinal
AUTHORS	Frankfurt Univ.	TAC group

*no experiment

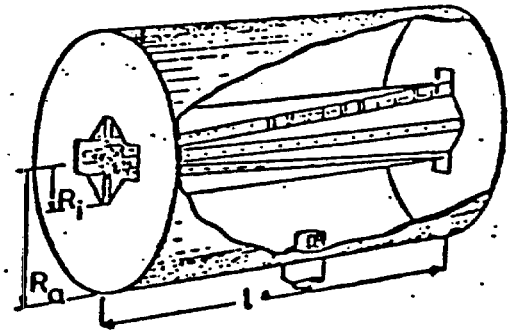
Up to date we can adjust longitudinal field flatness by various methods (Shapping. Shimming of vane edge or end tuning) only in static state but not in dynamic state. This is important for high intensity beam and high duty factor RFQ Linacs.



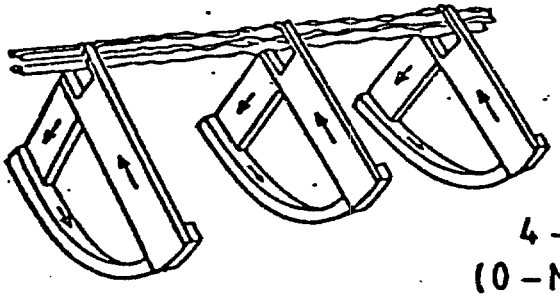
Four Vane Resonator
(TE_{210}-like)



Double-H-Resonator
($2 \cdot TE_{110}$-like)

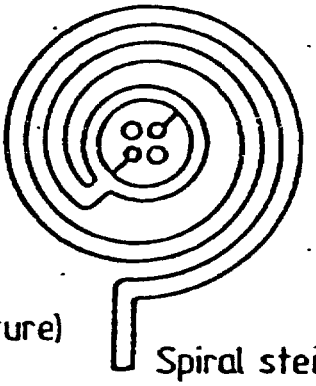


Split Coaxial Resonator (TEM-like)

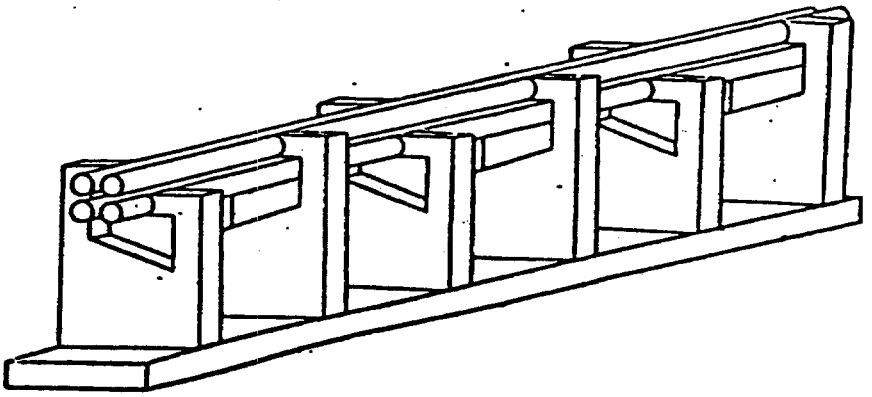


Straight stems

4-ROD-RFQ
(0-Mode-$\lambda/2$ structure)



Spiral stems



Linear 4-ROD-RFQ

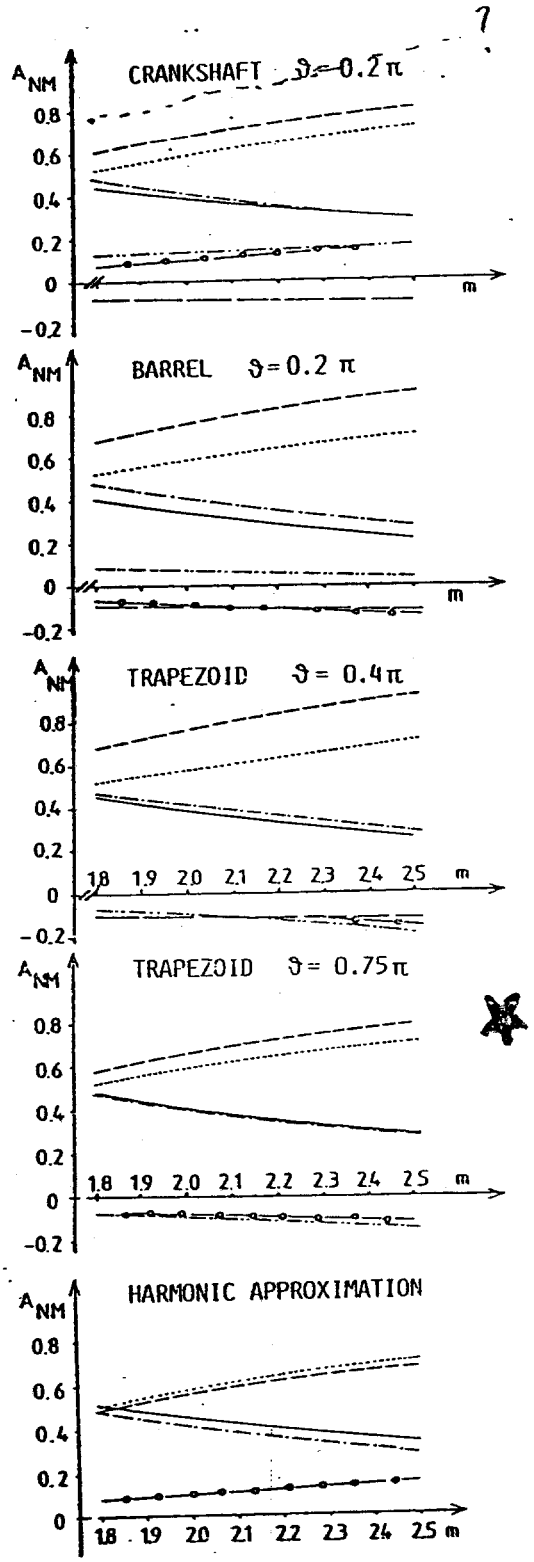


Fig. 5 Field harmonics A_{NM} of representative section with ka = 0.274 versus modulation (a = 3 mm, 108 MHz, 300 keV proton energy)
——— A_{10}, ····· ideal A_{10},
—·—·— ideal A_{01}, ——— A_{01},
—··—··— A_{12}, —•—•— A_{21},
——— A_{30}
Normalization $A_{01} = A_{01}/a^2$

$A_{NM} = A_{NM}/I_{2M}\ (Nk\frac{a+b}{2})$

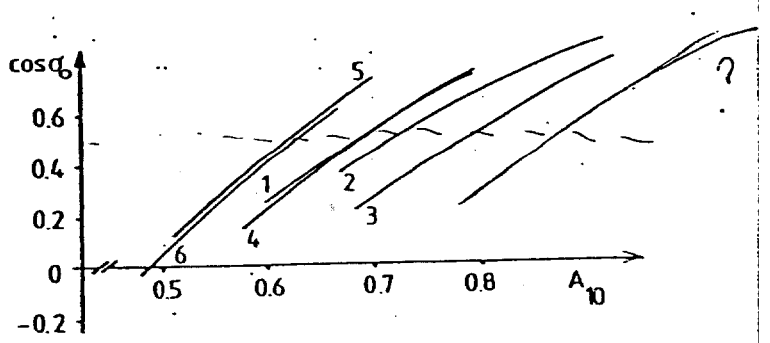


Fig. 6 $\cos\sigma_0$ versus A_{10} of figs. 5, electrode voltage 30 kV
1 crankshaft, 2 barrel, 3 trapezoid $\vartheta = 0.4\pi$, 4 trapezoid $\vartheta = 0.75\pi$, 5 ideal, 6 approximation of 5

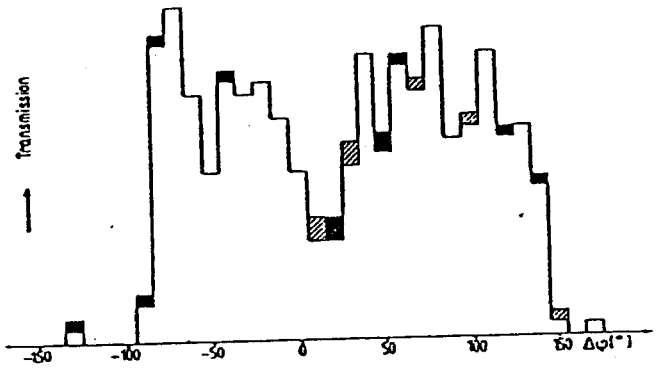


Fig. 7 Gains ▬ and losses ▨ of particles due to A_{21} in proton linac[3,4,5]. Transmission about 6 mA, shaper omitted in design, PARMTEQ code, transmission versus phase plotted.

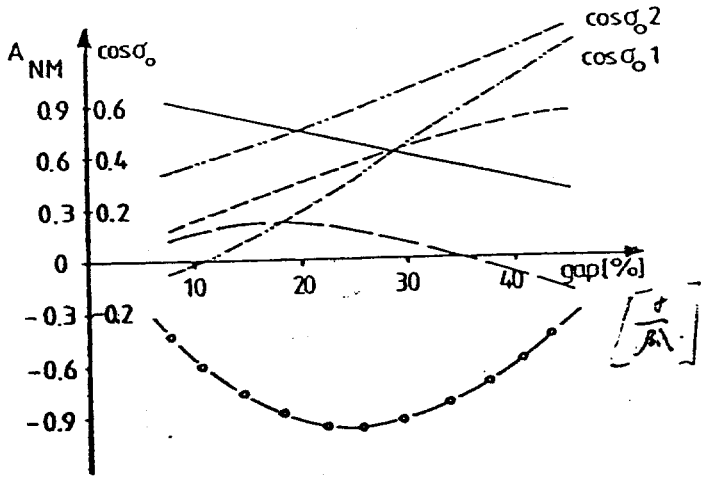


Fig. 8 Field harmonics of fingers type 4 with ka = 0.274 and $\cos\sigma_0$ versus gap in % of $\beta\lambda$, a = 6 mm, 13.5 MHz, 18.75 keV/amu energy, $\cos\sigma_0$ corresponding to
1. 120 kV for ions with τ/amu = 1/130
resp. 2. 150 kV, 1/208
Legend of curves as in fig. 5

5. Limit and Problem

If we look at two term potential field

$$U = \frac{V}{2}\left[X\left(\frac{r}{a}\right)^2 \cos 2\vartheta + A I_0(Kr)\cos Kz\right] \qquad (5.1)$$

$$E_r = -\frac{XV}{a^2} r\cos 2\vartheta - \frac{KAV}{2} I_1(Kr)\cos Kz \qquad (5.2)$$

$$E_\vartheta = \frac{XV}{a^2} r \sin 2\vartheta \qquad (5.3)$$

$$E_z = \frac{KAV}{2} I_0(Kr)\sin Kz \qquad (5.4)$$

$$A = \frac{m^2 - 1}{m^2 I_0(Ka) + I_0(mKa)} \qquad (5.5)$$

$$X = 1 - A I_0(Ka) \qquad (5.6)$$

In a design of RFQ A should increase so $m \rightarrow$ increase, $X \rightarrow$ decrease

A – acceleration efficiency

X – focusing efficiency

In formula (5.2) in order to keep E_r constant radius of aperture a have to b decreased. So further increase A is limited. In our example design 200 MHz, $W_f = 2.0$ MeV, $E_z \rightarrow$ decrease at output end. (see Fig6.)

In order to build a short RFQ A must increase, that means more effective high gradient RFQ structure should be invented. (or chose high rf frequency from design point of view)

The field stabization in longitudinal direction need more study for four-vane RFQ structure.

(24)

DESIGN ,FREQ=201.25 MHZ, Q=1.0,WI0.030,WF= 2.00 AMU= 1.01 I= PAGE 1

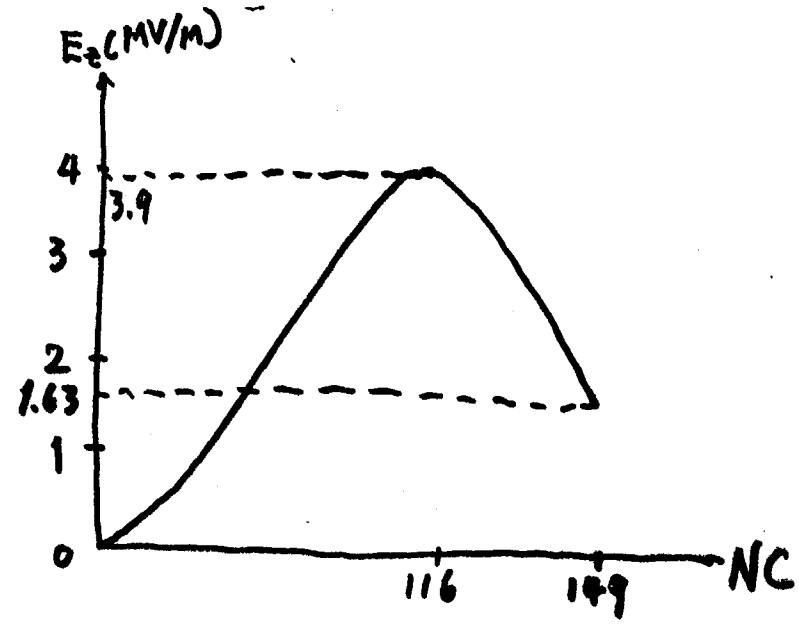
TANK 1 LENGTH= 204.84 CM, 149 CELLS, CHARGE STATE 1.

NC	V	WS	BETA	EZ	CAPA	PHI	A	M	B	CL	TL
00	.119	0.03000	.0080	0.000	0.00000	-90.0	2.218	1.000	0.572		
10	.119	0.03000	.0080	0.000	0.00000	-90.0	1.207	1.000	1.930	0.596	0.60
20	.119	0.03000	.0080	0.000	0.00000	-90.0	0.925	1.000	3.289	0.596	1.19
30	.119	0.03000	.0080	0.000	0.00000	-90.0	0.778	1.000	4.647	0.596	1.79
40	.119	0.03000	.0080	0.000	0.00000	-90.0	0.685	1.000	6.006	0.596	2.38
50	.119	0.03000	.0080	0.000	0.00000	-90.0	0.618	1.000	7.364	0.596	2.98
60	.119	0.03000	.0080	0.000	0.00000	-90.0	0.568	1.000	8.723	0.596	3.57
70	.119	0.03000	.0080	0.000	0.00000	-90.0	0.528	1.000	10.081	0.596	4.17
80	.119	0.03000	.0080	0.000	0.00000	-90.0	0.496	1.000	11.440	0.596	4.76
90	.119	0.03000	.0080	0.014	0.00068	-89.9	0.495	1.002	11.440	0.596	5.36
100	.119	0.03000	.0080	0.027	0.00136	-89.8	0.495	1.005	11.440	0.596	5.96
110	.119	0.03000	.0080	0.041	0.00205	-89.7	0.494	1.007	11.440	0.596	6.55
120	.119	0.03000	.0080	0.054	0.00272	-89.7	0.494	1.010	11.441	0.596	7.15
130	.119	0.03010	.0080	0.068	0.00340	-89.6	0.493	1.012	11.440	0.596	7.74
140	.119	0.03010	.0080	0.081	0.00408	-89.5	0.492	1.015	11.440	0.596	8.34
150	.119	0.03010	.0080	0.095	0.00475	-89.4	0.492	1.017	11.440	0.596	8.94
160	.119	0.03010	.0080	0.108	0.00542	-89.3	0.491	1.020	11.440	0.596	9.53
170	.119	0.03010	.0080	0.122	0.00609	-89.2	0.491	1.022	11.440	0.596	10.13
180	.119	0.03010	.0080	0.135	0.00676	-89.1	0.490	1.025	11.440	0.596	10.72
190	.119	0.03020	.0080	0.148	0.00743	-89.1	0.490	1.027	11.440	0.596	11.32
200	.119	0.03020	.0080	0.162	0.00810	-89.0	0.489	1.030	11.440	0.596	11.92
210	.119	0.03020	.0080	0.175	0.00877	-88.9	0.488	1.032	11.440	0.597	12.51
220	.119	0.03020	.0080	0.188	0.00944	-88.8	0.488	1.034	11.440	0.597	13.11
230	.119	0.03030	.0080	0.202	0.01011	-88.7	0.487	1.037	11.440	0.597	13.71
240	.119	0.03030	.0080	0.215	0.01078	-88.6	0.487	1.039	11.440	0.597	14.30
250	.119	0.03030	.0080	0.228	0.01144	-88.5	0.486	1.042	11.440	0.597	14.90
260	.119	0.03040	.0080	0.241	0.01211	-88.4	0.486	1.044	11.440	0.598	15.50
270	.119	0.03040	.0081	0.254	0.01276	-88.4	0.485	1.047	11.440	0.598	16.10
280	.119	0.03050	.0081	0.267	0.01342	-88.3	0.485	1.049	11.440	0.598	16.70
290	.119	0.03050	.0081	0.280	0.01408	-88.2	0.484	1.052	11.440	0.599	17.29
300	.119	0.03060	.0081	0.293	0.01474	-88.1	0.483	1.054	11.440	0.599	17.89
310	.119	0.03060	.0081	0.306	0.01540	-88.0	0.483	1.056	11.440	0.599	18.49
320	.119	0.03070	.0081	0.319	0.01607	-87.9	0.482	1.059	11.440	0.600	19.09
330	.119	0.03080	.0081	0.332	0.01674	-87.8	0.482	1.061	11.440	0.601	19.69
340	.119	0.03080	.0081	0.345	0.01741	-87.8	0.481	1.064	11.440	0.601	20.29
350	.119	0.03090	.0081	0.358	0.01809	-87.7	0.481	1.066	11.440	0.602	20.90
360	.119	0.03100	.0081	0.371	0.01878	-87.6	0.480	1.068	11.440	0.602	21.50
370	.119	0.03110	.0081	0.384	0.01946	-87.5	0.480	1.071	11.440	0.603	22.10
380	.119	0.03120	.0082	0.397	0.02016	-87.4	0.479	1.073	11.440	0.604	22.71
390	.119	0.03130	.0082	0.410	0.02086	-87.3	0.479	1.076	11.440	0.605	23.31
400	.119	0.03140	.0082	0.424	0.02157	-87.2	0.478	1.078	11.440	0.606	23.92
410	.119	0.03150	.0082	0.437	0.02228	-87.1	0.478	1.081	11.440	0.607	24.52
420	.119	0.03160	.0082	0.450	0.02300	-87.1	0.477	1.083	11.440	0.608	25.13
430	.119	0.03170	.0082	0.463	0.02371	-87.0	0.477	1.085	11.440	0.609	25.74
440	.119	0.03190	.0082	0.475	0.02432	-86.9	0.476	1.087	11.440	0.610	26.35
450	.119	0.03200	.0083	0.486	0.02495	-86.8	0.476	1.089	11.440	0.611	26.96
460	.119	0.03210	.0083	0.497	0.02558	-86.7	0.476	1.091	11.440	0.612	27.57
470	.119	0.03230	.0083	0.508	0.02623	-86.6	0.475	1.093	11.440	0.614	28.19
480	.119	0.03250	.0083	0.520	0.02688	-86.5	0.475	1.095	11.440	0.615	28.80
490	.119	0.03260	.0083	0.532	0.02755	-86.4	0.474	1.097	11.440	0.617	29.42
500	.119	0.03280	.0084	0.543	0.02822	-86.3	0.474	1.099	11.440	0.618	30.04
510	.119	0.03300	.0084	0.555	0.02891	-86.3	0.474	1.102	11.440	0.620	30.66
520	.119	0.03320	.0084	0.567	0.02962	-86.2	0.473	1.104	11.440	0.622	31.28
530	.119	0.03340	.0084	0.579	0.03033	-86.1	0.473	1.106	11.440	0.623	31.90
540	.119	0.03360	.0085	0.591	0.03106	-86.0	0.472	1.108	11.440	0.625	32.53
550	.119	0.03380	.0085	0.603	0.03181	-85.9	0.472	1.110	11.440	0.627	33.15
560	.119	0.03410	.0085	0.616	0.03257	-85.8	0.472	1.112	11.440	0.629	33.78
570	.119	0.03430	.0086	0.628	0.03334	-85.7	0.471	1.114	11.440	0.631	34.41
580	.119	0.03460	.0086	0.641	0.03414	-85.6	0.471	1.116	11.441	0.634	35.05
590	.119	0.03480	.0086	0.654	0.03495	-85.5	0.470	1.118	11.441	0.636	35.68
600	.119	0.03510	.0086	0.665	0.03567	-85.4	0.470	1.120	11.441	0.638	36.32

One posibility to build a short RFQ Linac is to increase Kilpatrick limit and use high rf power source to get high field gradient (at TAC Using existing RFQ structure). The other posibility to build a short RFQ Linac is to seek new RFQ structure which has high accelerating eff. meanwhile keeps the focusing strength constant.

NC	V	WS	BETA	EZ	CAPA	PHI	A	M	B	CL	TL
610	.119	0.03540	.0087	0.674	0.03629	-85.3	0.470	1.121	11.441	0.641	36.96
620	.119	0.03570	.0087	0.683	0.03693	-85.3	0.470	1.122	11.441	0.643	37.61
630	.119	0.03600	.0088	0.692	0.03759	-85.2	0.469	1.124	11.441	0.646	38.25
640	.119	0.03630	.0088	0.702	0.03826	-85.1	0.469	1.125	11.441	0.649	38.90
650	.119	0.03660	.0088	0.711	0.03894	-85.0	0.469	1.126	11.441	0.652	39.55
660	.119	0.03700	.0089	0.721	0.03965	-84.9	0.468	1.127	11.441	0.655	40.21
670	.119	0.03730	.0089	0.730	0.04037	-84.8	0.468	1.129	11.441	0.658	40.87
680	.119	0.03770	.0090	0.740	0.04111	-84.7	0.468	1.130	11.441	0.661	41.53
690	.119	0.03810	.0090	0.750	0.04186	-84.6	0.468	1.131	11.441	0.664	42.19
700	.119	0.03850	.0091	0.760	0.04264	-84.5	0.467	1.133	11.441	0.667	42.86
710	.119	0.03890	.0091	0.770	0.04343	-84.4	0.467	1.134	11.441	0.671	43.53
720	.119	0.03930	.0092	0.781	0.04425	-84.3	0.467	1.136	11.441	0.675	44.20
730	.119	0.03980	.0092	0.791	0.04509	-84.2	0.467	1.137	11.441	0.678	44.88
740	.119	0.04020	.0093	0.802	0.04594	-84.1	0.466	1.138	11.441	0.682	45.56
750	.119	0.04070	.0093	0.812	0.04682	-84.0	0.466	1.140	11.441	0.686	46.25
760	.119	0.04120	.0094	0.823	0.04778	-83.6	0.466	1.141	11.441	0.691	46.94
770	.119	0.04170	.0094	0.832	0.04867	-83.1	0.465	1.142	11.441	0.696	47.64
780	.119	0.04230	.0095	0.842	0.04957	-82.6	0.465	1.143	11.441	0.701	48.34
790	.119	0.04300	.0096	0.852	0.05052	-82.1	0.465	1.144	11.441	0.706	49.04
800	.119	0.04370	.0096	0.862	0.05153	-81.6	0.465	1.145	11.441	0.711	49.75
810	.119	0.04440	.0097	0.872	0.05259	-81.1	0.465	1.146	11.441	0.717	50.47
820	.119	0.04530	.0098	0.883	0.05371	-80.6	0.464	1.147	11.441	0.724	51.19
830	.119	0.04620	.0099	0.894	0.05490	-80.1	0.464	1.148	11.441	0.731	51.93
840	.119	0.04710	.0100	0.905	0.05615	-79.5	0.464	1.149	11.441	0.738	52.66
850	.119	0.04820	.0101	0.917	0.05746	-79.0	0.463	1.151	11.441	0.746	53.41
860	.119	0.04930	.0102	0.928	0.05883	-78.5	0.463	1.152	11.441	0.754	54.16
870	.119	0.05040	.0104	0.940	0.06027	-77.9	0.463	1.153	11.441	0.763	54.93
880	.119	0.05170	.0105	0.952	0.06178	-77.4	0.463	1.154	11.441	0.773	55.70
890	.119	0.05310	.0106	0.964	0.06336	-76.8	0.462	1.155	11.441	0.782	56.48
900	.119	0.05450	.0108	0.976	0.06501	-76.2	0.462	1.157	11.441	0.793	57.28
910	.119	0.05610	.0109	0.988	0.06672	-75.7	0.462	1.158	11.441	0.804	58.08
920	.119	0.05770	.0111	0.999	0.06851	-75.1	0.461	1.159	11.441	0.816	58.89
930	.119	0.05950	.0113	1.011	0.07037	-74.5	0.461	1.160	11.441	0.828	59.72
940	.119	0.06140	.0114	1.023	0.07229	-73.9	0.461	1.162	11.441	0.841	60.56
950	.119	0.06340	.0116	1.047	0.07533	-73.0	0.460	1.165	11.441	0.856	61.42
960	.119	0.06570	.0118	1.078	0.07894	-71.8	0.459	1.169	11.441	0.872	62.29
970	.119	0.06830	.0121	1.108	0.08267	-70.7	0.458	1.174	11.441	0.888	63.18
980	.119	0.07110	.0123	1.139	0.08664	-69.5	0.457	1.179	11.441	0.905	64.08
990	.119	0.07420	.0126	1.170	0.09085	-68.4	0.456	1.184	11.441	0.924	65.01
1000	.119	0.07760	.0129	1.200	0.09530	-67.2	0.455	1.189	11.441	0.945	65.95
1010	.119	0.08130	.0132	1.230	0.09999	-65.9	0.454	1.194	11.441	0.967	66.92
1020	.119	0.08560	.0135	1.287	0.10734	-64.4	0.452	1.204	11.441	0.993	67.91
1030	.119	0.09040	.0139	1.344	0.11520	-62.9	0.450	1.214	11.441	1.020	68.93
1040	.119	0.09580	.0143	1.402	0.12356	-61.3	0.448	1.225	11.441	1.049	69.98
1050	.119	0.10200	.0147	1.463	0.13302	-59.6	0.445	1.237	11.440	1.082	71.06
1060	.119	0.10910	.0152	1.550	0.14567	-57.8	0.442	1.255	11.440	1.118	72.18
1070	.119	0.11720	.0158	1.635	0.15908	-55.9	0.438	1.273	11.440	1.158	73.34
1080	.119	0.12650	.0164	1.747	0.17655	-53.9	0.433	1.299	11.440	1.202	74.54
1090	.119	0.13750	.0171	1.867	0.19649	-51.8	0.427	1.328	11.440	1.252	75.79
1100	.119	0.15040	.0179	2.024	0.22243	-49.7	0.420	1.368	11.440	1.308	77.10
1110	.119	0.16570	.0188	2.202	0.25377	-47.4	0.411	1.418	11.441	1.371	78.47
1120	.119	0.18410	.0198	2.419	0.29336	-45.1	0.399	1.484	11.440	1.443	79.92
1130	.119	0.20650	.0210	2.685	0.34420	-42.6	0.384	1.577	11.440	1.525	81.44
1140	.119	0.23400	.0223	3.006	0.40922	-40.1	0.364	1.709	11.440	1.620	83.06
1150	.119	0.26840	.0239	3.425	0.49812	-37.5	0.335	1.924	11.440	1.731	84.79
1160	.119	0.31170	.0258	3.900	0.60907	-35.0	0.295	2.272	11.440	1.858	86.65
1170	.119	0.35970	.0277	3.717	0.61886	-34.9	0.293	2.272	11.440	1.976	88.63
1180	.119	0.40830	.0295	3.514	0.62449	-34.8	0.292	2.272	11.440	2.115	90.74
1190	.119	0.45730	.0312	3.339	0.63042	-34.7	0.291	2.272	11.440	2.247	92.99
1200	.119	0.50670	.0328	3.185	0.63515	-34.6	0.290	2.272	11.440	2.373	95.36
1210	.119	0.55640	.0344	3.050	0.63901	-34.5	0.289	2.272	11.440	2.493	97.85
1220	.119	0.60640	.0359	2.929	0.64221	-34.4	0.288	2.272	11.440	2.609	100.46
1230	.119	0.65660	.0374	2.821	0.64490	-34.3	0.288	2.272	11.440	2.720	103.18
1240	.119	0.70710	.0388	2.724	0.64720	-34.2	0.288	2.272	11.440	2.828	106.01
1250	.119	0.75770	.0402	2.635	0.64919	-34.0	0.287	2.272	11.440	2.931	108.94
1260	.119	0.80850	.0415	2.555	0.65092	-33.9	0.287	2.272	11.440	3.032	111.98

PARMTEQ result obtained at Fermilab computer.



the end of gental bunching section

26

NC V	WS	BETA	EZ	CAPA	PHI	A	M	B	CL	TL
1270.119	0.85960	.0428	2.481	0.65244	-33.8	0.287	2.272	11.440	3.130	115.11
1280.119	0.91070	.0440	2.412	0.65379	-33.6	0.286	2.272	11.440	3.225	118.33
1290.119	0.96210	.0453	2.349	0.65499	-33.5	0.286	2.272	11.440	3.318	121.65
1300.119	1.01360	.0464	2.291	0.65607	-33.4	0.286	2.272	11.440	3.408	125.06
1310.119	1.06520	.0476	2.230	0.65704	-33.2	0.286	2.272	11.440	3.498	128.56
1320.119	1.11700	.0488	2.185	0.65793	-33.1	0.286	2.272	11.440	3.583	132.13
1330.119	1.16890	.0499	2.138	0.65873	-32.9	0.286	2.272	11.440	3.667	135.80
1340.119	1.22090	.0510	2.093	0.65946	-32.7	0.285	2.272	11.440	3.750	139.55
1350.119	1.27310	.0520	2.050	0.66014	-32.6	0.285	2.272	11.440	3.831	143.38
1360.119	1.32550	.0531	2.011	0.66076	-32.4	0.285	2.272	11.440	3.911	147.29
1370.119	1.37790	.0541	1.973	0.66133	-32.2	0.285	2.272	11.440	3.989	151.28
1380.119	1.43050	.0552	1.937	0.66187	-32.1	0.285	2.272	11.440	4.066	155.35
1390.119	1.48320	.0562	1.903	0.66236	-31.9	0.285	2.272	11.440	4.142	159.49
1400.119	1.53610	.0572	1.871	0.66282	-31.7	0.285	2.272	11.439	4.217	163.71
1410.119	1.58900	.0581	1.840	0.66325	-31.5	0.285	2.272	11.439	4.290	168.00
1420.119	1.64210	.0591	1.810	0.66365	-31.4	0.285	2.272	11.439	4.362	172.36
1430.119	1.69540	.0600	1.782	0.66403	-31.2	0.285	2.272	11.439	4.434	176.79
1440.119	1.74870	.0610	1.755	0.66439	-31.0	0.285	2.272	11.439	4.504	181.30
1450.119	1.80220	.0619	1.730	0.66472	-30.8	0.284	2.272	11.439	4.574	185.87
1460.119	1.85580	.0628	1.705	0.66504	-30.6	0.284	2.272	11.439	4.642	190.51
1470.119	1.90960	.0637	1.681	0.66533	-30.4	0.284	2.272	11.439	4.710	195.22
1480.119	1.96340	.0646	1.658	0.66562	-30.2	0.284	2.272	11.439	4.777	200.00
1490.119	2.01740	.0655	1.636	0.66588	-30.0	0.284	2.272	11.439	4.843	204.84

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OUTPUT 2 -1 33 56 00 00 1 500
OUTPUT 2 -1 33 56 00 01 5 500 5
OUTPUT 4 1 33 56 33 01 1
SCHEFF   50.0 0.0497 0.0596 10 20 5 10
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      END
FORTRAN STOP
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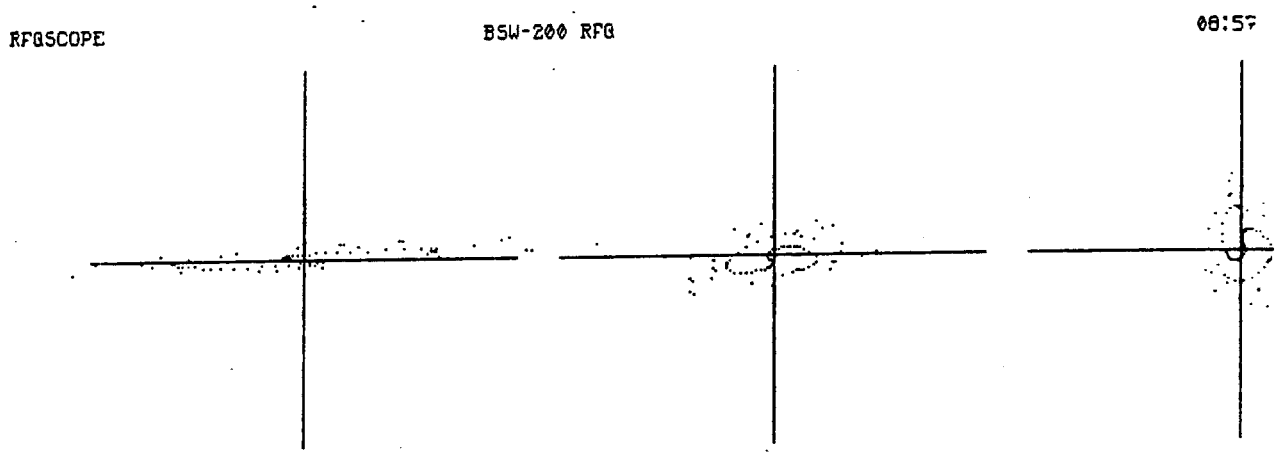

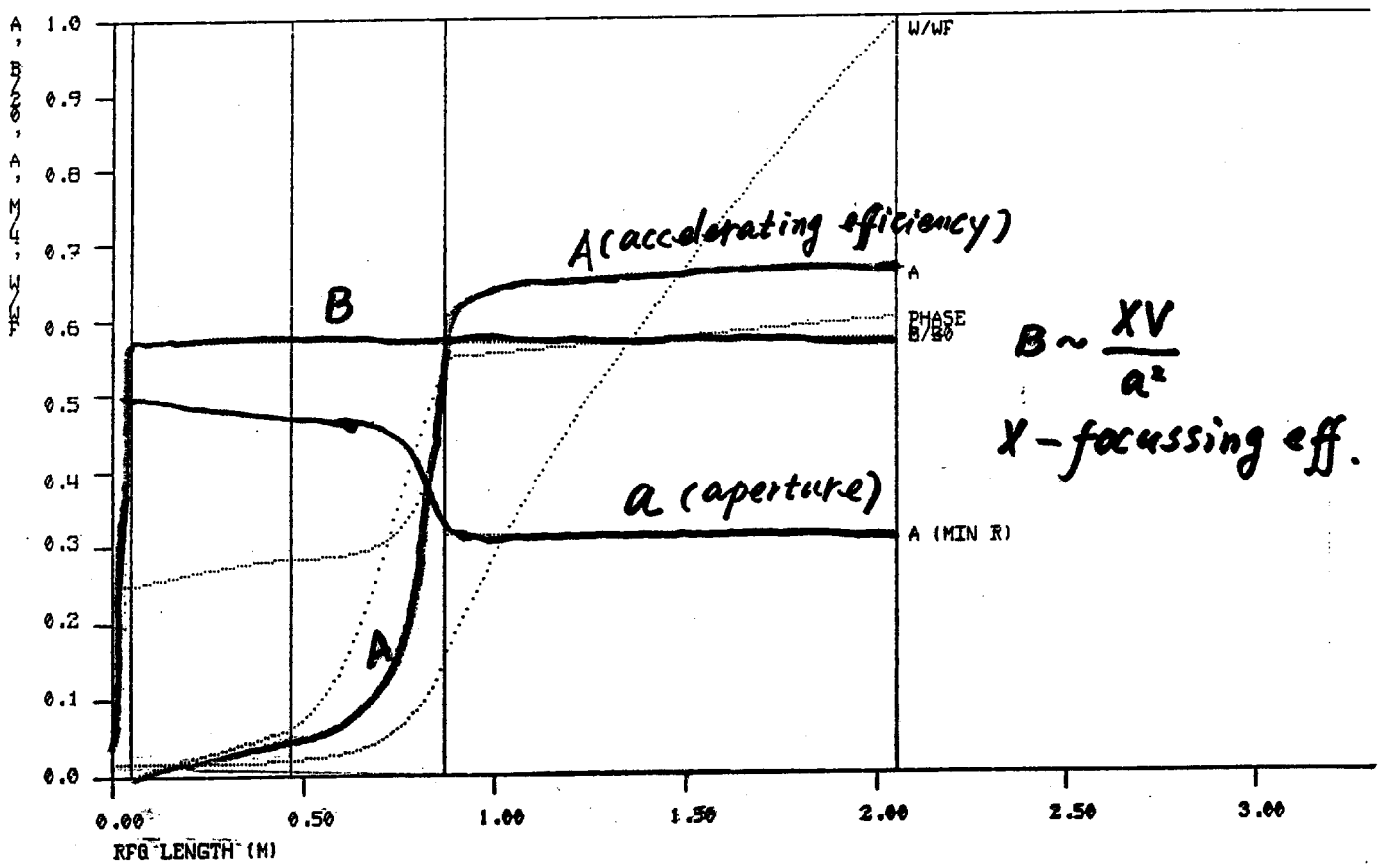

$B \sim \frac{XV}{a^2}$

X – focussing eff.