Introduction to Dabor Lenses - J. Peterson 9-24-87

1. Motivation – we would like to neutralize the space-charge forces which are increasing the emittance of the beam in the front end of the linac.

   a) 750 keV lines. There is emittance growth in each of the 750 keV lines. We know that space-chg. blowup and dilution of phase-space is a cause.

   b) RFQ injection. Energies 30-50 keV, $I \sim 50$ mA. The beam is "space-chg. dominated" ($\frac{KR^2}{\epsilon^2} \simeq 60 \gg 1$).

   Possible solution – a Dabor lens (D. Dabor, 1947)

2. Properties of a Dabor lens (to be shown):

   a) A Dabor lens contains a plasma which neutralizes the space chg of the beam

   b) Axial symmetry. Within the lens the fields have no dependence on the angle θ (cyl. coor. r, θ, z). $B_\theta = E_\theta = 0$. The focusing is due almost entirely to the radial electric field.

   b) Penning geometry – crossed E and B fields familiar from ion pump, Penning trap, magnetron, etc.

3. Theory

   Principal Investigators

D. Dabor	Nature 160, 89 (1947)
R. Booth + H. Lefevre	NIM 151, 143 (1978)
R. Mobley et. al.	'79 PAC
A. Morozov + S. Lebedev	Reviews of Plasma Physics, v. 8 (1980)

Plasma "fluid" equation of motion (electron comp.) Gaussian units

$$mn\frac{d\mathbb{v}}{dt} = -en(\mathbb{E} + \frac{1}{c}\mathbb{v}\times\mathbb{B}) - \vec{\nabla}p \qquad (1)$$

look for steady state solutions with $\frac{d\mathbb{v}}{dt} = 0$. If the plasma is "cold" (1 or 2 eV is typical for our application) a first approximation is obtained by neglecting ∇p term ($p = nkT$).

$$\Rightarrow \quad 0 = -en(\mathbb{E} + \frac{1}{c}\mathbb{v}\times\mathbb{B}) \Rightarrow \boxed{\mathbb{E} = -\frac{1}{c}\mathbb{v}\times\mathbb{B}} \qquad (2)$$

This equation has some important implications. Taking the cross-product of (2) with $\mathbb{B}$,

$$\mathbb{E}\times\mathbb{B} = \frac{1}{c}\mathbb{B}\times(\mathbb{v}_\perp\times\mathbb{B}) = \frac{1}{c}[\mathbb{v}_\perp B^2 - \mathbb{B}(\cancelto{0}{\mathbb{v}_\perp\cdot\mathbb{B}})] \quad ,$$

$$\mathbb{v}_\perp = c\left(\frac{\mathbb{E}\times\mathbb{B}}{B^2}\right) \quad . \qquad (3)$$

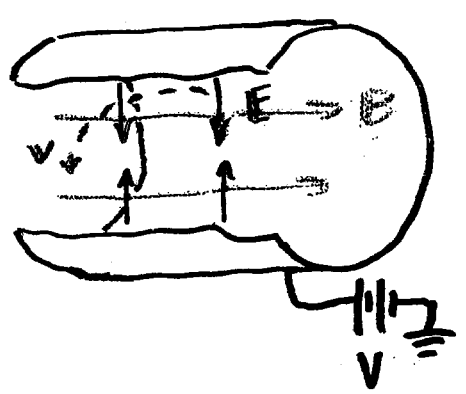
Now take the dot product of (2) with $d\vec{l}$, where $d\vec{l}$ lies along a magnetic field line.

$$\mathbb{E}\cdot d\vec{l} = -\nabla\varphi\cdot d\vec{l} = -d\varphi = 0 . \qquad (4)$$

Furthermore $\mathbb{E}\cdot\mathbb{v} = \mathbb{E}\cdot\mathbb{B} = 0$.

Thus, for a cylindrical system:

1) Electrons drift across flux tubes.
2) Mag. flux tubes are electric equipotential surfaces.
3) $\mathbb{E}\perp\mathbb{B}$.



Now let the plasma have a finite temperature. Assume the velocity is Maxwellian so that $p = nkT$. We also assume that T is constant along a line of magnetic force. Then (1) becomes

$$0 = -en(\mathbb{E} + \tfrac{1}{c}\mathbb{v}\times\mathbb{B}) - kT\nabla n \quad , \quad \mathbb{E} = -\vec{\nabla}\varphi$$

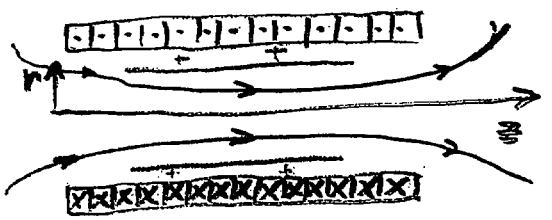
$$\mathbb{E}\cdot d\mathbb{l} = -d\varphi = -\frac{kT}{e}\left(\frac{dn}{n}\right)$$

integrate

$$\varphi = \varphi_0 + \frac{kT}{e}\ln\left(\frac{n}{n_0}\right) \qquad \text{(Boltzmann)}$$

In general the potential φ can vary along a flux tube.

Now consider the central region of a Gabor lens. Assume $n = \text{const.}$, cyl. coor., $E_z \ll E_r$. Poisson's eqn is



$$\nabla^2\phi = -4\pi e(n_i - n_e)$$

$$\frac{1}{r}\frac{d}{dr}\left(r\frac{d\phi}{dr}\right) = 4\pi e n$$

Integrate:

$$\phi = \phi_0 + \pi e n r^2$$

$$E_r = -\frac{d\phi}{dr} = -2\pi e n r \qquad \text{(focusing field)}$$

Suppose $\phi_0 = 0$, R = anode radius, U = anode potential $= \pi e n R^2$

$$n = \frac{U}{\pi e R^2} = 2.21\times10^6\,\frac{U}{R^2}$$

U in volts
R cm
n cm^{-3}

for neutralization we require

$n > n_{beam}$. Recall

$$J = \frac{I}{\pi a^2} = e n_b v \Rightarrow n_b = \frac{I}{\pi a^2 e v}$$

Equation of motion of positive ion (non-relativistic)

$$r'' + \left(\frac{u}{\varphi R^2}\right) r = 0$$

$$\varphi = \frac{\frac{1}{2} m v^2}{q^z} = \text{acc. potential of ions.}$$

$$\frac{1}{f} = \sqrt{K} \sin\sqrt{K} L \qquad K = \frac{u}{\varphi R^2} \qquad \text{Gabor}$$

$$K = \frac{B'}{(B\rho)_0} \qquad \text{Quad}$$

Numerical example: Gabor lens with $L = 30$ cm, $R = 4$ cm, $u = 5$ kV, $\varphi = 30$ kV, $I = 50$ mA, $a \sim 2$ cm

$$n_b \simeq 1 \times 10^8 \text{ cm}^{-3}$$

$$n = 7 \times 10^8 \text{ cm}^{-3} > n_b$$

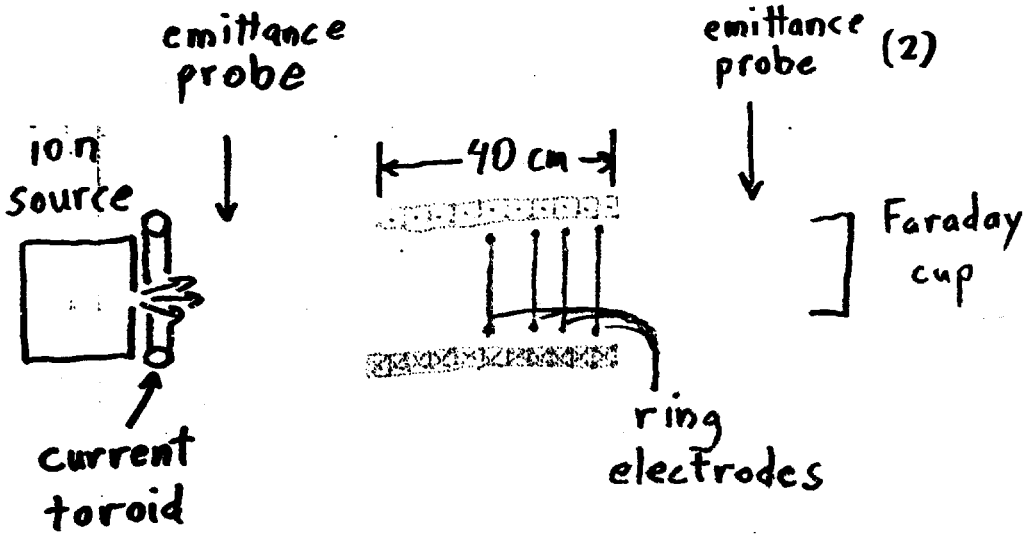
$$f = 19 \text{ cm}$$

Gabor lenses have been operated with positive ion beams. What about negative ions (e.g., H^-)?

In order to focus negative ions we need a plasma with a net positive chg. density. We will try 2 approaches on the test stand and see what works.

1. Start the lens with a neg. voltage on the electrodes. It may be necessary to add a hot cathode.

2. Start the lens with pos. voltage and pulse it negative.


emittance probe
emittance probe (2)
ion source
40 cm
Faraday cup
current toroid
ring electrodes

Conclusion:

The Gabor lens shows promise as a means for preventing emittance growth caused by space-charge forces in low-energy ion beams. Furthermore, it provides an axisymmetric focus which is desirable for matching into an RFQ.

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11-13-87

The RMS Emittance

P. M. Lapostolle, IEEE Trans. Nucl. Sci., 18, p. 1101 (1971)
also
C. Lejeune and J. Aubert in "Applied Charged Particle Optics," A. Septier, ed. (1980)

Consider a K-V distribution in x-x' trace-space, a so-called "perfect" beam. $f(x,x') = 1$ at all pts. on or inside the ellipse

$$\epsilon = \gamma x^2 + 2\alpha x x' + \beta x'^2,$$
$$\gamma\beta - \alpha^2 = 1,$$

$f = 0$ otherwise.

Average values are defined in the usual way

$$\langle \phi \rangle = \frac{1}{N} \iint f(x,x')\, \phi \, dx\, dx',$$

$$N = \iint f(x,x')\, dx\, dx'$$

Now calculate the 2nd moments of the perfect beam. We find

$$\langle x^2 \rangle = \frac{\epsilon\beta}{4} \qquad N = \pi \cdot \epsilon = \text{area}.$$

$$\langle x'^2 \rangle = \frac{\epsilon\gamma}{4}$$

$$\langle x x' \rangle = -\frac{\epsilon\alpha}{4}$$

Now observe that

$$\langle x^2\rangle\langle x'^2\rangle - \langle xx'\rangle^2 = \frac{\epsilon^2}{16}(\gamma\beta - \alpha^2) = \frac{\epsilon^2}{16}$$

thus

$$\epsilon_{rms} = 4\sqrt{\langle x^2\rangle\langle x'^2\rangle - \langle xx'\rangle^2} \qquad (\pi \text{ mm}\cdot\text{mrad})$$

for the perfect beam.

For non-ideal (real) beams, one <u>defines</u> the rms emittance using this result. In a Gaussian beam, 86.5% of the particles are contained by the rms emittance. Thus with each beam, we associate an "equivalent perfect beam," having same current and 2nd moments.

Advantages of rms emittance:

1. There is not a question of beam fraction
2. The rms emittance is an invariant of the motion if and only if the focusing forces are linear (Teng, 1971).
3. It is a good measure of beam quality.
4. It is useful in studying beams in which space-charge forces are important.

   and

5. It gives us a prescription for fitting an ellipse (α, β, γ) to a real beam.

emittance data 11/12/87

Normalized rms emittance, mm mrad	0.49	0.46	0.5
alpha	-2.31	-0.827	-2.69
beta, m	0.385	0.162	0.424
gamma, 1/m	16.5	10.4	19.4
current, ma	31.8	25.7	33
kinetic energy, keV	45	45	45

Normalized rms emittance $= \gamma\beta\,\epsilon_{rms}$, where

$$\gamma\beta = \frac{v}{\sqrt{c^2 - v^2}}$$

What we measure: $I(x, x')$.

Computer (Masscomp) evaluates $\langle x\rangle$, $\langle x'\rangle$, $\langle x^2\rangle$, $\langle x'^2\rangle$, $\langle xx'\rangle$, $\sigma_{xx} = \langle x^2\rangle - \langle x\rangle^2$, $\sigma_{x'x'}$, $\sigma_{xx'} = \langle xx'\rangle - \langle x\rangle\langle x'\rangle$. Then calculate

$$\epsilon_{rms} = 4\sqrt{\sigma_{xx}\sigma_{x'x'} - \sigma_{xx'}^2}, \quad \gamma = \frac{\epsilon_{rms}\sigma_{x'x'}}{4}, \quad \beta = \frac{\epsilon_{rms}\sigma_{xx}}{4},$$

$$\alpha = -\frac{\epsilon_{rms}\sigma_{xx'}}{4}$$