Coupled Cavity Structures for the Linac Upgrade

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Linac Upgrade: Increase final linac energy from 200 MeV to 400 MeV by replacing last 4 Alvarez drift-tube tanks with more efficient, higher gradient structure.

Accel. grad. ~ 3 x present grad. ~ 7.5 MV/m.

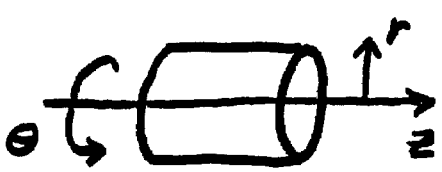
$P/L = (E_0 T)^2 / ZT^2$ (T = transit time factor)

Want high effective shunt impedance ZT^2

e.g. $P/L = 1\ MW/m$, $E_0 T = 7.5\ MV/m \times 0.8 \Rightarrow ZT^2 = 36\ M\Omega/m$

Linac's are "coupled cavity" structures in which many connected cavities are powered by one rf source.

Single cavity modes: ω_{mnp}



TM_{mnp} (no H_z)

TE_{mnp} (no E_z)

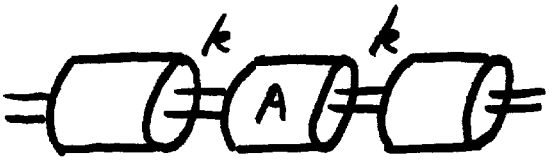
m = # full period azimuthal variations 0, 1, 2, ...

n = # half period radial variations 1, 2, ...

p = # half period axial variations 0, 1, 2, ...

Nagle, Knapp + Knapp (1967):

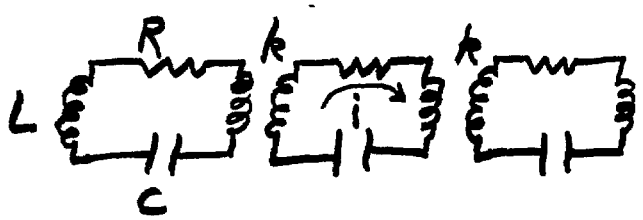
Describe properties of coupled chain using separate modes of single cavity as they develop into "bands" of coupled system.



Coupled cavities

$Q = \omega U/P = \omega/\Delta\omega$

$Z = E_0^2/P/L$



Coupled circuits

Linear lattice

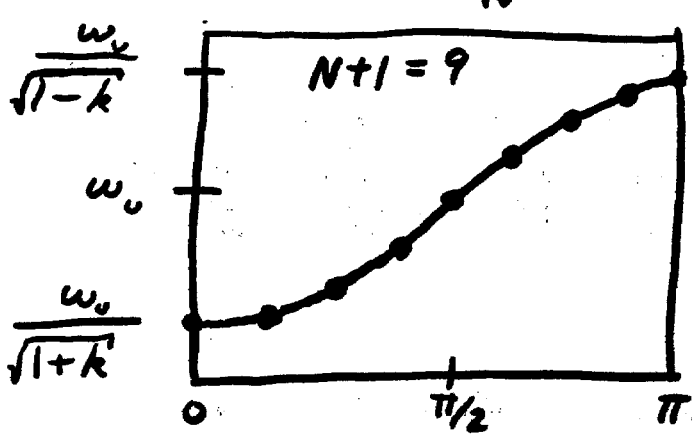
For a chain of $N+1$ cavities, each single cavity mode yields $N+1$ normal modes ($Q \to \infty$)

$$A_n^{(q)} = A_0 \cos \frac{\pi q n}{N} e^{i\omega_q t}$$

mode: $q = 0, 1, \ldots N$

$$\omega_q^2 = \frac{\omega_0^2}{1 + k \cos \frac{\pi q}{N}}$$

cell: $n = 0, 1, \ldots N$



Mode spacing

k/N^2 near $0, \pi$

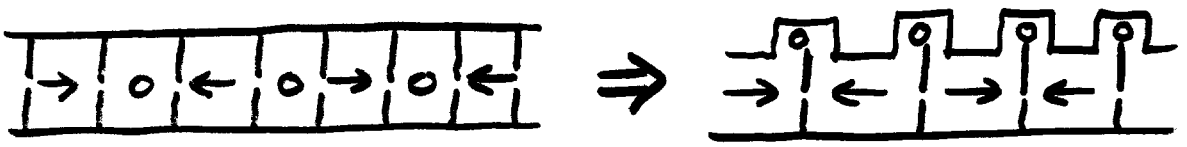
k/N near $\frac{\pi}{2}$

phase shift per cell

$\pi/2$ mode has special properties useful for long accelerating structures. This mode is generally insensitive to perturbations.

1. $\frac{\pi}{2}$ mode has field in cell n ($Q \to \infty$)

$$A_n^{(N/2)} = A_0 \cos \frac{n\pi}{2} \, e^{i\omega_0 t}$$



Optimize "accel. cells" for high shunt impedance. Q_c of "coupling cells" not very important.

2. Losses due to finite Q excite small amplitudes ($1/kQ_c$) in coupling cells and introduce amplitude droop ($1/(k^2 Q_a Q_c)$) in accel. cells. Losses do <u>not</u> introduce phase shift between accel. cells up to order $1/(kQ)^2$ if all cells tuned to same frequency.
   → "phase stabilized structure"

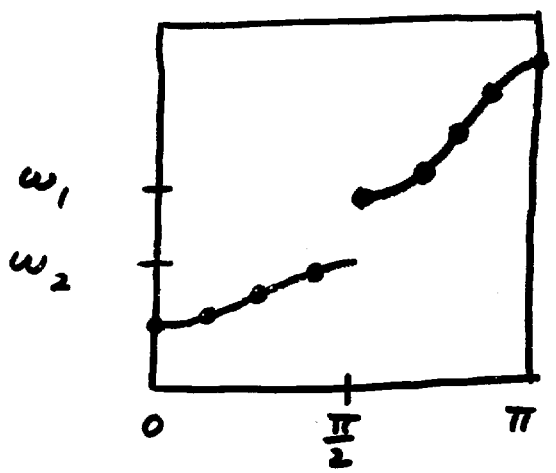
3. In $\frac{\pi}{2}$ mode, frequency errors $\Delta\omega_n$ in cells of chain produce only second order ($\Delta\omega_n \Delta\omega_m$) amplitude variations in accel. cells. → "amplitude stabilized"

Extension of model to more complicated structure

2N+1 total cells

$$k^2\cos^2\phi = \left(1 - \frac{\omega_a^2}{\omega^2} + k_a\cos 2\phi\right)\left(1 - \frac{\omega_c^2}{\omega^2} + k_c\cos 2\phi\right)$$

$$\phi = \frac{\pi q}{2N}, \quad q = 0, 1, \ldots 2N$$



$$\omega_1^2 = \frac{\omega_a^2}{1-k_a}$$

$$\omega_2^2 = \frac{\omega_c^2}{1-k_c}$$

To operate in $\frac{\pi}{2}$ mode requires closed stopband,

$$\omega_1 = \omega_2$$

Practical quantities of interest:

1. Power-flow droop for finite Q

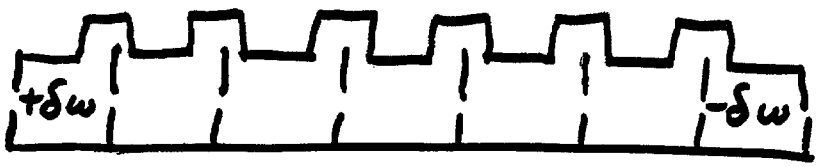
$$\left|\frac{\delta A}{A}\right| = \frac{2M^2\sqrt{(1-k_a)(1-k_c)}}{k^2 Q_a Q_c}$$

M = # accel. cells between two points

2. Power-flow phase shift for finite stopband

$$\delta\phi = \frac{4M^2(1-k_c)\sqrt{1-k_a}}{k^2 Q_a} \frac{\delta\omega_s}{\langle\omega_{\pi/2}\rangle}$$

3. Tilt-sensitivity parameter (for comparing tuning error sensitivity of different structures)



$$\frac{|\Delta A/A|}{\delta\omega/\langle\omega_{\pi/2}\rangle} = \frac{16M(1-k_a)(1-k_c)}{k^2} \frac{\delta\omega_s}{\langle\omega_{\pi/2}\rangle}$$

These quantities depend on Q's and coupling constants, k_i. The Q's are obtained by measuring the width of a resonance.

The coupling constants are typically found by measuring the mode spectrum (ω vs. phase shift) and doing least-squares fit using the dispersion relation.

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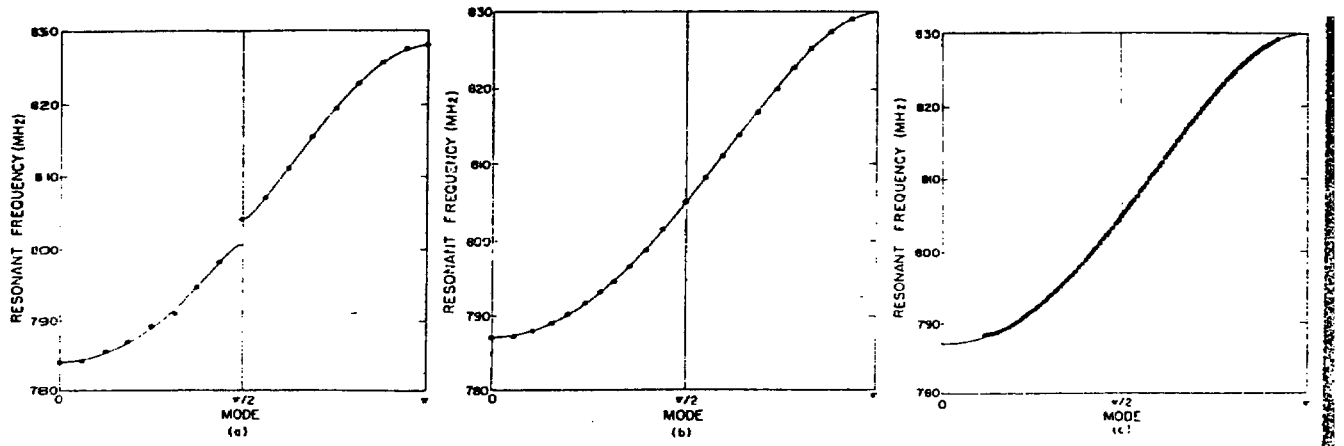


FIG. 8. (a) Dispersion curve obtained for a short section of model K before the stopband was closed. The solid curve is a least squares fit of Eq. (9) to the experimental points. (b) Dispersion curve for a short section of model K after the stopband had been closed. (c) Dispersion curve for the completely assembled model K tank.

the observed behavior of actual accelerator tanks may be demonstrated by comparing the observed mode spectra taken from accelerator tanks with fits to the data using this equation. Figure 8(a) shows a mode spectrum taken during the construction of model K, when a stopband still existed because the frequencies were not accurately set. Figure 8(b) shows the resultant curve for several cavities of the model K chain after the cavity frequencies had been adjusted to close the stopband. Figure 8(c) shows the dispersion curve for the assembled 119 cavity model K chain. Mistunings destroy the resonances in the region of the zero and π mode for a cavity chain as long as model K. The center region is still accurately described by the theory. The agreement of the calculated frequencies with the experimental data is remarkable. Even though we are violating the boundary conditions by terminating the chain in full cells rather than half cells, fits of the data to $\sim$1 part in 10^4 are common.

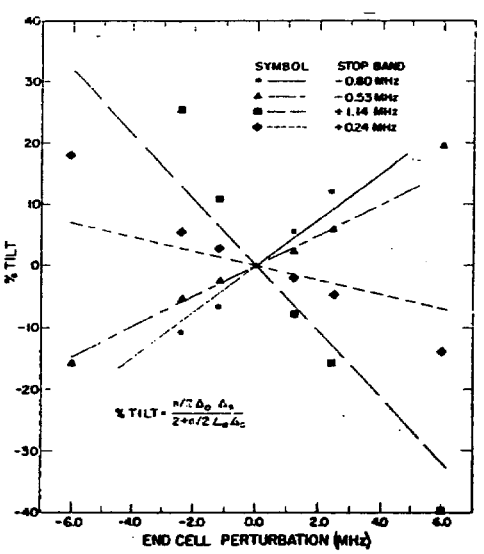


FIG. 9. Field tilt as a function of end cavity perturbations and stopband for 12 cells of model K. The curves are derived from Eq. (19); the points are corresponding experimental data.

The interpretation of the eigenfunctions X_n as square roots of stored energies has also been verified experimentally. In a two half cell plus one coupling cell configuration, perturbation measurements show the zero and π modes have twice the stored energy of the $\pi/2$ mode for a given axial field amplitude. This is true to quite high precision ($\sim$5%), even though the geometry of the coupling cavities is radically different from that of the accelerating cavity.

The coupling strengths are determined from the dispersion curve by least square fitting the measured points to Eq. (9). The slot length is empirically adjusted to give about 5% coupling, which is adequate for the cavity chains we are using. The coupling strength k_1 varies approximately as the slot length squared.

CAVITY CHAIN FIELD DISTRIBUTIONS

If the predictions of the coupled-resonator model are correct, rather large errors can be tolerated in the manufacturing of the cavities with little effect on performance. In Eq. (15) the error term is indicated to be of the form $\Delta_a\Delta_c$, in general rather complex sums over possible combinations of terms of this type. An estimate of the tolerance to errors may be made from the complete expression derivable from stepwise solution of the resonator equations and yields $\langle\Delta X/X\rangle_{rms}\cong N\langle\Delta_a\rangle_{rms}\cdot\langle\Delta_c\rangle_{rms}$ where the errors are assumed random and the effect of the sums is lumped in a statistical factor N. The assumption is made that no stopband is present. For $\langle\Delta_a\rangle=\langle\Delta_c\rangle=10^{-2}(\Delta\omega/\omega\sim10^{-4}$ for typical structures listed in Table I) $\langle\Delta X/X\rangle_{rms}\sim10^{-4}N$ or for a 100 cell tank we would expect $\sim$1% field fluctuations.

Model K SCS $\beta = .65$

*** COUPLING CONSTANTS OF ACCELERATING CAVITY ***

(1) CAVITY PARAMETERS

NUMBER OF MODES WITH PHIS/ACCEL. CELL IN [0,PI]: N = 11
ACCELERATING MODE FREQUENCY: FO(MHZ) = 8.05000E+02
NEAREST NEIGHBOR COUPLING CONSTANT: K = 4.94E-02
DIRECT COUPLING BETWEEN ACCEL. CELLS: KA = -1.28E-02
DIRECT COUPLING BETWEEN COUPL. CELLS: KC = 5.91E-03

(2) FITTING SUMMARY

NSMAX = 200
NSTEPS = 175
SUM OF SQR(NORMALIZED FREQ DIFF)/(N-2) = 2.35E-07

(3) MODE SPECTRUM (MHZ)

PHIS/(PI/N-1)	FA(CALC)	FA(EXPT)	FC(CALC)	FC(EXPT)
0	8.30000E+02	8.30000E+02	7.87151E+02	7.87151E+02
1	8.29607E+02	8.29337E+02	7.87363E+02	7.87312E+02
2	8.28452E+02	8.28163E+02	7.87992E+02	7.87849E+02
3	8.26611E+02	8.26429E+02	7.89022E+02	7.88925E+02
4	8.24192E+02	8.23878E+02	7.90427E+02	7.90267E+02
5	8.21333E+02	8.21071E+02	7.92175E+02	7.91882E+02
6	8.18176E+02	8.17849E+02	7.94233E+02	7.93763E+02
7	8.14858E+02	8.14462E+02	7.96572E+02	7.96183E+02
8	8.11497E+02	8.11129E+02	7.99163E+02	7.98871E+02
9	8.08188E+02	8.08172E+02	8.01981E+02	8.01828E+02
10	8.05000E+02	8.05000E+02	8.05000E+02	8.05000E+02

*** FITTING ROUTINE COMPLETED ***
14. 45. 30. UCLP, GG, TACF, 0.098KLNS. ** END OF LISTING **

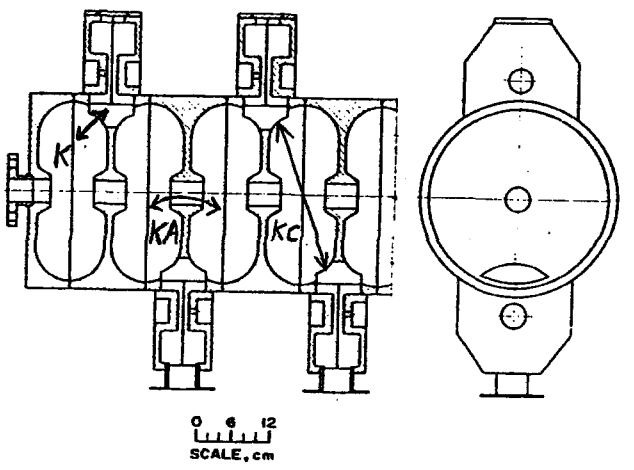
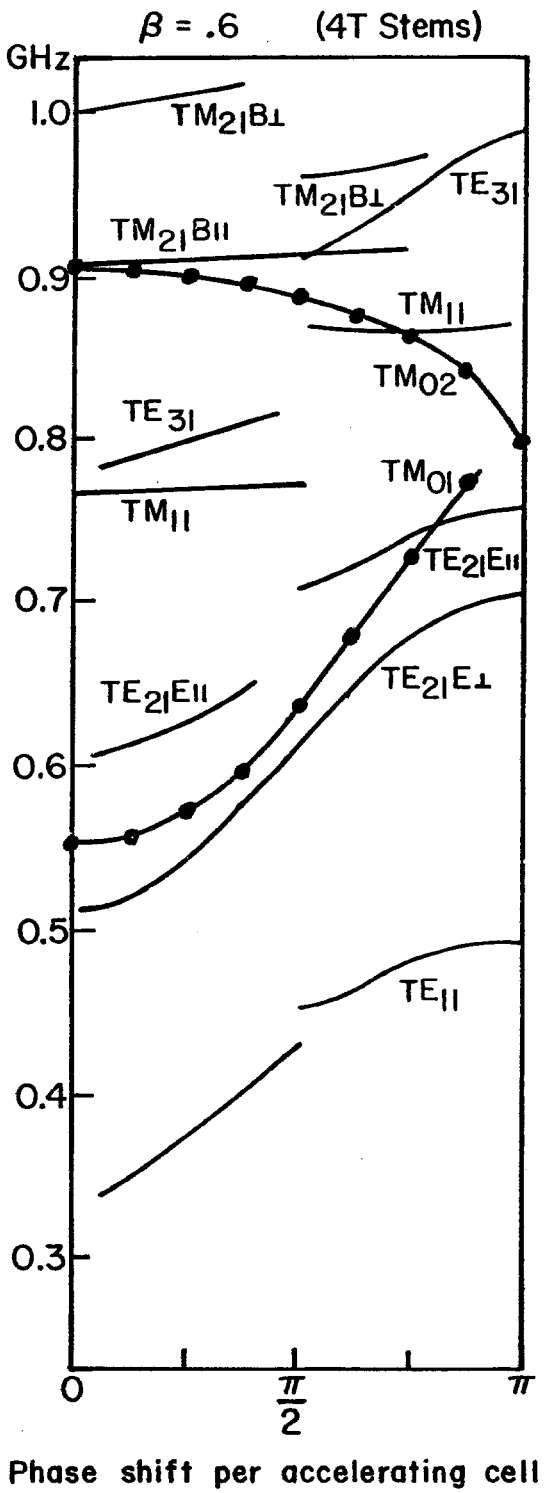


FIG. 7. A cross-sectional view of model K. The coupling slots are formed by the intersection of the right circular cylindrical shape of the coupling cavity and the spherically-shaped accelerating cavity.



Mode spectra for $\beta = 0.6$
DAW structure with 4T stems.

4T DAW $\beta = .6$

*** COUPLING CONSTANTS OF ACCELERATING CAVITY ***

(1) CAVITY PARAMETERS

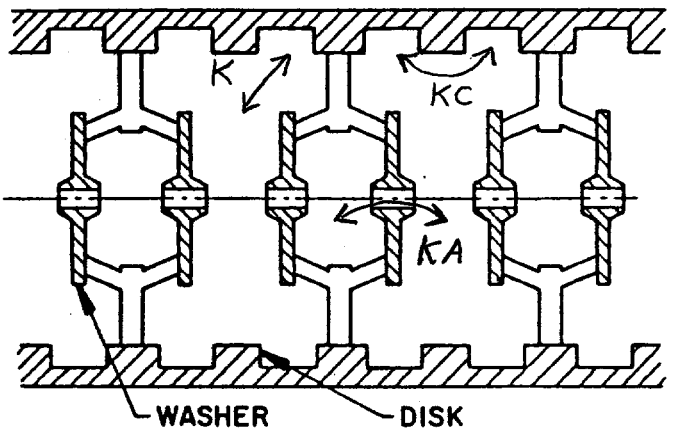
NUMBER OF MODES WITH PHIS/ACCEL. CELL IN [0,PI]: N = 9
ACCELERATING MODE FREQUENCY: FO(MHZ) = 8.05000E+02
NN COUPLING CONSTANT: K = 3.80E-01
NNN COUPLING CONSTANTS: KA = 3.02E-01
KC = -2.46E-02

(2) FITTING SUMMARY

NSMAX = 200
NSTEPS = 110
SUM OF SQR(NORMALIZED FREQ DIFF)/N = 3.58E-04

(3) MODE SPECTRUM (MHZ)

PHIS/(PI/N-1)	FA(CALC)	FA(EXPT)	FC(CALC)	FC(EXPT)
0	9.18600E+02	9.18600E+02	5.62500E+02	5.62500E+02
1	9.16892E+02	9.16300E+02	5.67217E+02	5.66200E+02
2	9.11871E+02	9.11500E+02	5.81383E+02	5.81100E+02
3	9.03773E+02	9.07800E+02	6.04939E+02	6.08800E+02
4	8.92758E+02	9.00700E+02	6.37414E+02	6.45000E+02
5	8.78606E+02	8.90200E+02	6.77353E+02	6.94400E+02
6	8.60398E+02	8.75700E+02	7.21709E+02	7.40400E+02
7	8.36451E+02	8.54400E+02	7.65852E+02	7.83400E+02
8	8.05000E+02	8.05000E+02	8.05000E+02	8.05000E+02

*** FITTING ROUTINE COMPLETED ***
07.28.49.UCLP, GG, TAOF, 0.094KLNS. ** END OF LISTING **

DISK AND WASHER