LOT 12/3/87

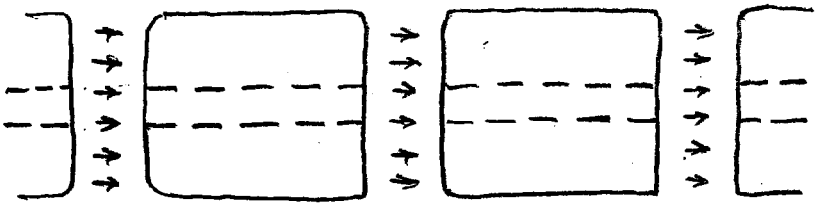
(A) Conceptual evolution of RFQ structures

(B) Examination of field patterns

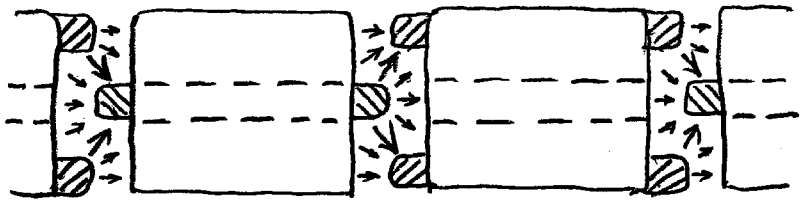
(C) Partition of focusing actions — relation to acceleration rate

(D) Transit time factor — adding high harmonics

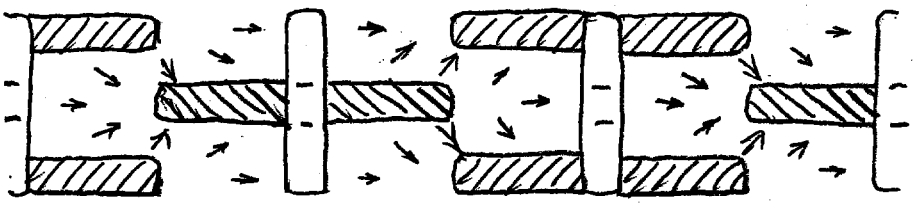
(A) Evolution

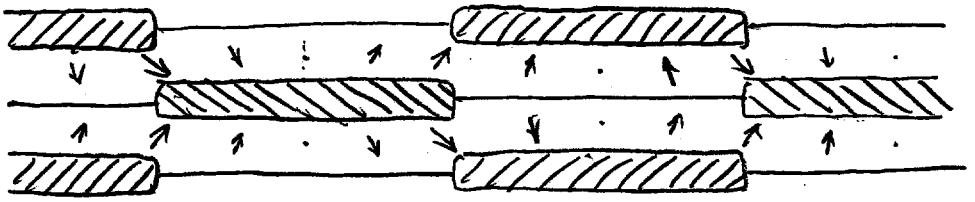


Alvarez (DTW)

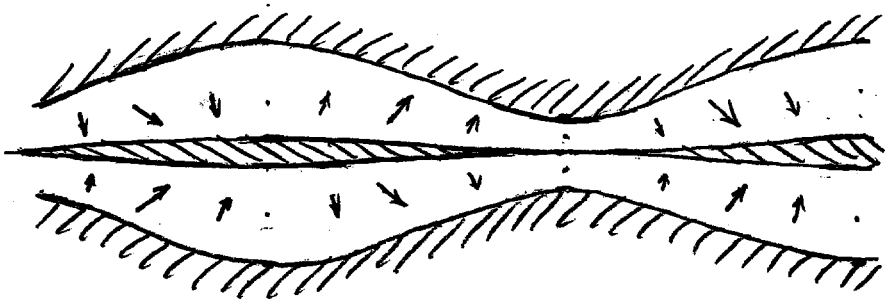


Vladimirski (DTN)





(ROD)



Kapchinski Taplyakov (VANE)

(A) Conceptual evolution of RFQ

(B) Partition of focusing action — relation to acceleration rate

(C) Transit-time factor — adding high harmonics

(B) Field Patterns

Electric potential (non-relativistic limit)

$$\Phi = U(r,\psi,z)\sin\omega t \qquad \text{(Kapchinski)}$$

$$U(r,\psi,z) = \underset{\substack{\uparrow \\ z\text{-independent}}}{F_0(r,\psi)} + \sum_{n=1}{}' F_n(r,\psi)\sin nkz \qquad k=\frac{2\pi}{\ell},\ k_n = k\sqrt{n^2-\beta^2}$$

$$\begin{cases} F_0(r,\psi) = \sum_{s=1}{}' A_{0s}\, r^{2s}\cos 2s\psi \\ F_n(r,\psi) = \sum_{s=0}{}'' A_{ns}\, I_{2s}(k_n r)\cos 2s\psi \end{cases}$$

(1) Cylindrically symmetric term (Smith–Gluckstern) $(s=0)$

$$\begin{cases} F_0(r,\psi) = 0 \\ F_n(r,\psi) = A_{n0}\, I_0(k_n r) \end{cases}$$

$$\Phi = \sum_{n=1}' A_{n0} I_0(k_n r) \sin nkz \sin \omega t \quad (5\text{-}6)$$

$$\begin{cases} E_z = \dfrac{\partial U}{\partial z} = \sum E_n I_0(k_n r) \cos nkz \sin \omega t \\ E_r = \dfrac{\partial U}{\partial r} = \sum \dfrac{nk}{k_n} E_n I_1(k_n r) \sin nkz \sin \omega t \\ B_\theta \quad = \sum \dfrac{\omega/c}{k_n} E_n I_1(k_n r) \cos nkz \cos \omega t \end{cases}$$

Only the fundamental harmonic ($n=1$) has secular effects

$$\Phi = A_{10} I_0(k_1 r) \sin kz \sin \omega t \qquad k_1 = k\sqrt{1-\beta^2} = \frac{k}{\gamma}$$

$$\begin{cases} E_z = E_1 I_0(k_1 r) \cos kz \sin \omega t \\ \quad \cong E_1 \frac{1}{2}\left[\sin(\omega t - kz) + \sin(\omega t + kz)\right] \qquad r \to 0 \\ \qquad\qquad \text{forward travelling} \qquad \text{backward travelling} \\ E_r = \frac{k}{k_1} E_1 I_1(k_1 r) \sin kz \sin \omega t \\ \quad = k E_1 r \frac{1}{2}\left[\cos(\omega t - kz) - \cos(\omega t + kz)\right] \qquad r \to 0 \\ B_\theta = \end{cases}$$

(2) Non-cylindrically symmetric (up to quadrupole term $\cos 2\psi$) and only fundamental terms

$$\begin{cases} F_0(r,\psi) = A_{01} r^2 \cos 2\psi = A_{01}(x^2 - y^2) \\ F_{\substack{n=1 \\ s=0}}(r,\psi) = A_{10} I_0(k_1 r) \end{cases}$$

$$\Phi = \left[A_{01}(x^2-y^2) + A_{10} I_0(k_1 r) \sin kz\right] \sin \omega t$$

$$= V\left[B\frac{x^2-y^2}{a^2} + A I_0(k_1 r) \sin kz\right] \sin \omega t$$

a = min. aperture (Vane-type, LANL 2)

<u>Vane contour</u> $\quad I_0(k_1 r) \cong 1$

$$V\left(B\frac{x^2-y^2}{a^2} + A \sin kz\right) = \pm V$$

xz: $\quad B\frac{x^2}{a^2} = 1 - A \sin kz \quad (+V) \quad B = 1 - A$

yz: $\quad B\frac{y^2}{a^2} = 1 + A \sin kz \quad (-V) \quad A < 1$

xy: $\quad B\frac{y^2-x^2}{a^2} = A \sin kz \mp 1$

$$\boxed{\Phi = V\left[\frac{1 - A I_0(k_1 a)}{a^2}(x^2-y^2) + A I_0(k_1 r) \sin kz\right] \sin \omega t}$$

A = amplitude of ripple

(c) Partition of Focusing

(1) General Theorem — 1st order focusing (Irrelevant for AG)

$$\nabla \cdot E = \frac{\partial E_x}{\partial x} + \frac{\partial E_y}{\partial y} + \frac{\partial E_z}{\partial z} = 0$$

$$m\ddot{x} = eE_x \cong e\frac{\partial E_x}{\partial x}x \equiv K_x x \qquad (NR)$$

etc.

force const.

Hence

$$K_x + K_y + K_{\Delta z} = 0 \qquad \left(\begin{array}{c}\Delta z \text{ relative to} \\ \text{moving centroid}\end{array}\right)$$

(2) Acceleration

On-axis

$$E_z = 2E_0 \cos kz \sin \omega t$$

$$= E_0\left[\sin(\omega t - kz) + \sin(\omega t + kz)\right]$$

drop

$$\frac{\partial E_z}{\partial z} = -kE_0 \cos(\omega t - kz)$$

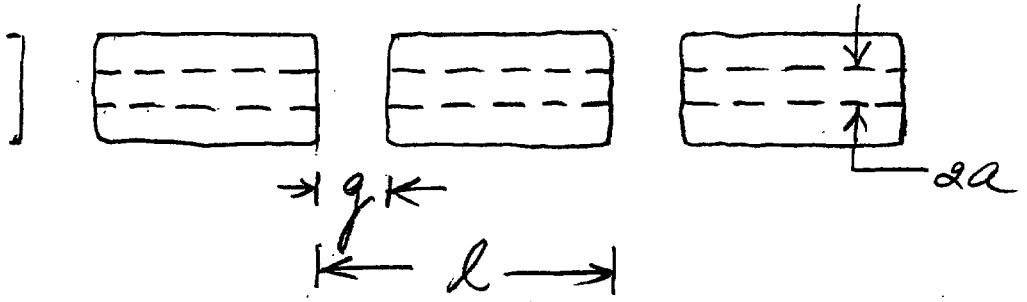
$$= (-k\cot\phi)E_z \qquad \phi = \omega t - kz_p = \text{const.}$$

$$\therefore \quad K_{\Delta z} = e\frac{\partial E_z}{\partial z} = (-k\cot\phi)\mathcal{E}' \qquad (\mathcal{E} = \text{energy})$$

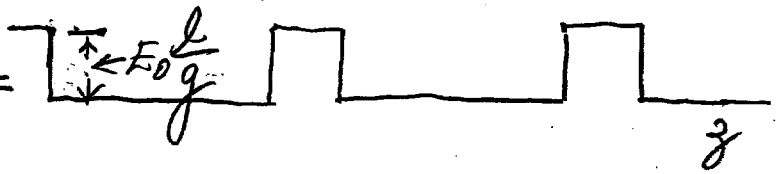
$$\boxed{K_x + K_z = (k\cot\phi)\mathcal{E}'}$$

(D) Transit Time Factor

(1) Circularly symmetric field — add harmonics to match the B.C. for DTL



B.C.

$E_z(r=a) =$ 

$\times \sin \omega t$

Result (Smith & Gluckstern)

$$E_z(r,z) = (2T) E_0 I_0(k_1 r) \cos kz \sin \omega t$$

$$\Rightarrow T E_0 I_0(k_1 r) \sin(\omega t - kz)$$

$$T = \frac{1}{I_0(k_1 a)} \frac{\sin \pi g/l}{\pi g/l} = \text{on-axis transit time factor}$$

g/l :	1	$\frac{1}{2}$	$\frac{1}{4}$	0
T :	0	0.64	0.90	1

Compared to only fundamental

$$E_z(r,z) = E_0 I_0(k_1 r) \cos kz \sin \omega t$$

$$\Rightarrow \frac{1}{2} E_0 I_0(k_1 r) \sin(\omega t - kz)$$

(Equivalent to $T = \frac{1}{2}$)

(T makes the opposite travelling wave partially useful)

(2) With quadrupole term (guesses)

$$\Phi = V\left[\left\{1-2TAI_0(k,a)\right\}\frac{x^2-y^2}{a^2} + 2TAI_0(k,r)\sin kz\right]\sin\omega t$$

(a) One gets the quad. coeff. as long as at the min. aperture a the highest potentials are $\pm V$.

(b) Bosi's structure approximates the DTN structure.

(c) Localizing quad. term in gap improves AG focusing. δ-functions ~ 2x sine function.

(d) In extreme case of $\frac{g}{l} \to 0$ $(T=1)$, and hence δ-function quad.

Focusing ~ $2(1-2A) = 1/2$
Accel. ~ $2A = 3/4$ at $A = \frac{3}{8}$

Compared to vanes

Focusing ~ $1-A = 1/2$
Accel. ~ $A = 1/2$ at $A = 1/2$