national accelerator laboratory

TM-277
0300

WAVE PROPAGATION BETWEEN BOOSTER LAMINATIONS INDUCED BY LONGITUDINAL MOTION OF BEAM

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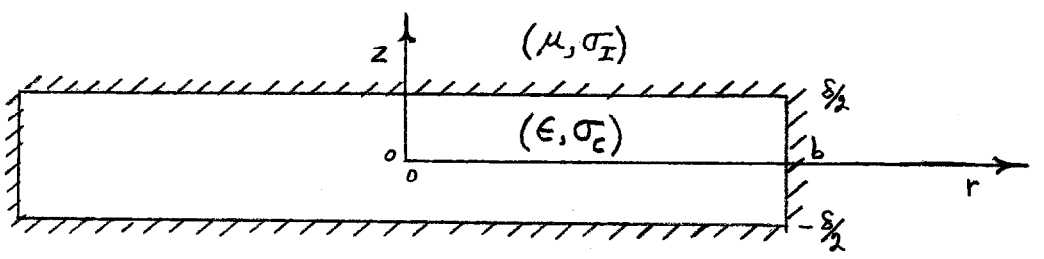
PURPOSE

To calculate the electromagnetic wave propagation in the crack between booster magnet laminations induced by longitudinal charge oscillations of the beam. The result is to be expressed in terms of a wave impedance at the iron boundary so that the effects of the propagation may be readily incorporated into beam dynamics calculations. Cylindrical geometry is assumed for simplicity.

RADIAL CRACK

An exact solution for an electromagnetic wave propagating radially in the geometry shown is to be found.



The transverse magnetic mode has its vector potential expressed as

$$\vec{A} = \nabla \times \vec{L} U \qquad (1)$$

where $\vec{L}$ is the operator[1]

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$$\vec{L} = \vec{k} x \nabla, \tag{2}$$

and $\vec{k}$ is the unit vector along the x-axis. Assuming a time variation of $e^{-i\,\omega t}$, one has

$$\vec{A} = \vec{k}\nabla^2 U - \nabla\frac{\partial U}{\partial z}, \qquad \vec{B} = -\vec{L}\nabla^2 U \tag{3}$$

For U, one has

$$\nabla^2 U_c + \left(\varepsilon\,\frac{\omega^2}{c^2} + \frac{4\pi\sigma_c\omega}{c^2}\,i\right) U_c = 0 \text{ (crack, gaussian)} \tag{4}$$

and

$$\nabla^2 U_I + \frac{4\pi\mu\sigma_I\omega}{c^2}\,i\,U_I = 0. \text{ (Iron, gaussian)} \tag{5}$$

Thus no displacement current is assumed in the iron and unit permeability is assumed in the crack.

The following may be taken as solutions to Eqs. (4) and (5).

$$U_c = C\,e^{-\lambda_I\frac{\delta}{2}}\,\mathrm{ch}\lambda_c z\,R_o(kr) \tag{6}$$

$$U_I = D\,\mathrm{ch}\lambda_c\frac{\delta}{2}\,e^{-\lambda_I z}\,R_o(kr) \qquad (z>0) \tag{7}$$

where

$$R_o(kr) = H_o^{(2)}(kr)H_o^{(1)}(kb) - \eta H_o^{(1)}(kr)H_o^{(2)}(kb). \tag{8}$$

In Eq. (8) the functions are Hankel functions, η is the reflection coefficient and k is the radial propagation constant. In order to satisfy Eqs. (4) and (5)

$$\lambda_c^2 = k^2 - \varepsilon\,\frac{\omega^2}{c^2} - \frac{4\pi\sigma_c\omega}{c^2}\,i \tag{9}$$

$$\lambda_I^2 = k^2 - \frac{4\pi\mu\sigma_I\omega}{c^2} i \tag{10}$$

For z>0 the fields are

$$E_{rc} = \frac{i\omega k\lambda_c}{c} C e^{-\lambda_I \frac{\delta}{2}} sh\lambda_c z R_1(kr) \tag{11}$$

$$E_{zc} = -\frac{i\omega k^2}{c} C e^{-\lambda_I \frac{\delta}{2}} ch\lambda_c z R_o(kr) \tag{12}$$

$$H_{\theta c} = -\frac{k\omega^2}{c^2} \left(\varepsilon + \frac{4\pi\sigma_c}{\omega} i\right) C e^{-\lambda_I \frac{\delta}{2}} ch\lambda_c z R_1(kr) \tag{13}$$

$$E_{rI} = -\frac{i\omega k\lambda_I}{c} D ch\lambda_c \frac{\delta}{2} e^{-\lambda_I z} R_1(kr) \tag{14}$$

$$E_{zI} = -\frac{i\omega k^2}{c} D ch\lambda_c \frac{\delta}{2} e^{-\lambda_I z} R_o(kr) \tag{15}$$

$$H_{\theta I} = -\frac{4\pi\sigma_I\omega k i}{c^2} D ch\lambda_c \frac{\delta}{2} e^{-\lambda_I z} R_1(kr) , \tag{16}$$

where

$$R_1(kr) = H_1^{(2)}(kr)H_o^{(1)}(kb) - \eta H_1^{(1)}(kr)H_o^{(2)}(kb). \tag{17}$$

Boundary conditions at $z = \delta/2$ require continuity of E_r and H_θ. This gives

$$\lambda_c C + \lambda_I \coth\left(\lambda_c \frac{\delta}{2}\right) D = 0 \tag{18}$$

$$\omega \left(\varepsilon + \frac{4\pi\sigma_c}{\omega} i\right) C - 4\pi i\sigma_I D = 0, \tag{19}$$

Equations (9), (10), (18), and (19) are solved simultaneously for the radial propagation constant k. The condition is

$$\frac{4\pi\sigma_I}{\omega} i \sqrt{\frac{c^2k^2}{\omega^2} - \varepsilon - \frac{4\pi\sigma_c}{\omega} i}$$

$$+ \left(\varepsilon + \frac{4\pi\sigma_c}{\omega} i\right) \sqrt{\frac{c^2k^2}{\omega^2} - \frac{4\pi\mu\sigma_I}{\omega} i} \quad .$$

$$\coth \left(\frac{\omega\delta}{2c} \sqrt{\frac{c^2k^2}{\omega^2} - \varepsilon - \frac{4\pi\sigma_c}{\omega} i}\right) = 0 \tag{20}$$

The wave impedance in the crack will be calculated only for z = 0 since the crack is assumed quite small. Then,

$$Z(r) = - \frac{E_{zc}(r,o)}{H_{\theta c}(r,o)} \quad . \tag{21}$$

To determine the reflection coefficient η, set the wave impedance in the crack at r = b equal to the wave impedance for a plane wave in iron. Thus

$$Z(b) = (1 - i) \sqrt{\frac{\mu\omega}{8\pi\sigma_I}} \quad . \tag{22}$$

Equations (12)and (13) then yield

$$\eta = \frac{\frac{ck}{\omega} H_o^{(2)}(kb) + \left(\varepsilon + \frac{4\pi\sigma_c}{\omega} i\right)\sqrt{\frac{i\mu\omega}{4\pi\sigma_I}}\, H_1^{(2)}(kb)}{\frac{ck}{\omega} H_o^{(1)}(kb) + \left(\varepsilon + \frac{4\pi\sigma_c}{\omega} i\right)\sqrt{\frac{i\mu\omega}{4\pi\sigma_I}}\, H_1^{(1)}(kb)} \cdot \frac{H_o^{(1)}(kb)}{H_o^{(2)}(kb)} \quad (23)$$

and

$$Z(a) = -\frac{ick}{\varepsilon\omega + 4\pi\sigma_c i} \cdot \frac{R_o(ka)}{R_1(ka)} \quad (24)$$

RESULTS

Table 1 lists the permeability, permittivity and conductivity of materials that seem appropriate for booster laminations. The dielectric properties of core plating could not be obtained from the manufacturer. It has been assumed that they are close to a phosphate glass. Manufacturer's technical literature generally lists permeabilities as a function of frequency for the thickness of sheet manufactured. To obtain the incremental permeability ($\mu' + i\ u''$), one needs measurements on very small spheres in order to be free from flux exclusion by eddy currents at the higher frequencies. This data was taken from the FERROTRON[2] design manual which lists the permeability of 3 micron diameter pure iron spheres uniformly imbedded in an insulating plastic binder. The permeability is sensibly constant up to 1000 MHz indicating that the imaginary component is quite small. ARMCO Thin Electrical Steels[3] indicate that the incremental permeability for 4 mil

sheet may be about 100 at 400 Hz for high excitation (bias), and low incremental induction. Combining the two bits of information, it seems reasonable to take the incremental permeability as 100 with no imaginary component in the frequency range of interest.

TABLE 1. Assumed Properties of Materials

	μ'	μ''	ε'	ε''	σ(Hz)
Iron	100	-	-	-	4.5×10^{16}
Core Plate	-	-	-	-	9×10^{6}
Epoxy[4]	-	-	3.8	-	9×10^{-2}
Phosphate Glass[4]	-	-	5.2	-	-

TABLE 2. Assumed Dimension

a(in)	b(in)	D_{lamin}(in)	δ_{crack}(in)
.750	6.000	.0250	.000375

To obtain values of the propagation constant k and impedances Z, the permeability of the iron was taken to be $\mu = 100$, the permittivity of the crack was taken to be a mixture of 1/3 that of epoxy with 2/3 that of phosphate glass. The conductivity of iron is taken to be 20 μohm-cm and that of the crack to be 10^5 ohm-cm.

Table 3 presents the results from the program CRACK. The propagation constant k is given in units of ω/c. In the impedance columns, first the real part is presented and then

the imaginary. Gaussian units are employed, hence all impedances are dimensionless. By bore impedance, the impedance of Eq. (22) is meant. Crack impedance is the value given in Eq. (24). Guide impedance is series combination of bore impedance and crack impedance each taken in proportion to its relative thickness along the inner bore of the magnet. Figure 1 presents a plot of the guide impedance as a function of frequency.

In particular, it is to be noted from Table 3 that the crack impedance introduces a large resistive component into the resultant guide impedance which would equal the bore impedance for zero crack width.

REFERENCES

[1] S. C. Snowdon, Properties of the L-Operator, MURA Technical Note TN-506 (Oct. 19, 1964).

[2] FERROTRON Design Manual, The Polymer Corp. Engineered Products Division, Reading, Penna. (1963); see also: GAF Carbonyl Iron Powders, General Aniline and Film Corporation, 140 W. 51 Street, New York, New York 10020 (1960).

[3] ARMCO Thin Electrical Steels, Armco Steel Corporation, Middletown, Ohio (1963).

[4] Dielectric Materials and Applications, A. R. VonHippel, Editor, The MIT Press, Cambridge, Mass. (1954).

TABLE 1. Propagation Constant and Impedances

RADIAL CRACK PROPAGATION CONSTANT AND IMPEDANCE

FREQ MIN (MHZ) =	10.00000	FREQ MAX (MHZ) =	500.00000	NFREQ =	50
CONDUCT+16/SEC) =	4.50000	PERMEABILITY =	100.00000	AIR DIELECTRIC =	4.75000
CRACK WIDTH (IN) =	.00038	LAMINATION THICK (IN) =	.02500	INNER RAD (IN) =	.75000
OUTER RAD (IN) =	6.00000	CRACK COND (MHZ) =	9.00000		

TRANSVERSE MAGNETIC MODE

FREQ (MHZ)	PROPAGATION CONSTANT		BORE IMPEDANCE		CRACK IMPEDANCE		GUIDE IMPEDANCE		AN
10.000	1.95556E+01	1.25479E+01	1.62447E-04	-1.62447E-04	6.63047E-01	-6.08556E-01	9.95877E-03	-9.15348E-03	.00000
20.000	1.59162E+01	8.84192E+00	2.29734E-04	-2.29734E-04	1.05945E+00	-8.19505E-01	1.58832E-02	-1.23373E-02	.00000
30.000	1.54733E+01	7.47044E+00	2.81366E-04	-2.81366E-04	1.48992E+00	-8.56175E-01	2.22957E-02	-1.29300E-02	.00000
40.000	1.45001E+01	6.70835E+00	3.24893E-04	-3.24893E-04	1.87369E+00	-6.43574E-01	2.80100E-02	-9.83104E-03	.00000
50.000	1.37770E+01	6.20446E+00	3.63242E-04	-3.63242E-04	1.99625E+00	-2.62832E-01	2.98591E-02	-4.24208E-03	.00000
60.000	1.32078E+01	5.83763E+00	3.97911E-04	-3.97911E-04	1.85141E+00	1.90905E-03	2.77528E-02	-3.63818E-04	.00000
70.000	1.27422E+01	5.55384E+00	4.29793E-04	-4.29793E-04	1.66334E+00	5.98215E-02	2.50094E-02	4.60620E-04	.00000
80.000	1.23505E+01	5.32494E+00	4.59468E-04	-4.59468E-04	1.55026E+00	8.86445E-03	2.33630E-02	-3.21676E-04	.00000
90.000	1.20140E+01	5.13465E+00	4.87340E-04	-4.87340E-04	1.51473E+00	-6.75375E-02	2.28655E-02	-1.47823E-03	.00000
100.000	1.17201E+01	4.97271E+00	5.13701E-04	-5.13701E-04	1.53187E+00	-1.31855E-01	2.31446E-02	-2.45471E-03	.00000
110.000	1.14601E+01	4.83246E+00	5.38774E-04	-5.38774E-04	1.57881E+00	-1.70315E-01	2.38629E-02	-3.04778E-03	.00000
120.000	1.12275E+01	4.70919E+00	5.62731E-04	-5.62731E-04	1.63756E+00	-1.79525E-01	2.47548E-02	-3.20749E-03	.00000
130.000	1.10175E+01	4.59957E+00	5.85709E-04	-5.85709E-04	1.69391E+00	-1.61907E-01	2.56101E-02	-2.96976E-03	.00000
140.000	1.08265E+01	4.50115E+00	6.07819E-04	-6.07819E-04	1.73743E+00	-1.24197E-01	2.62752E-02	-2.43425E-03	.00000
150.000	1.06516E+01	4.41192E+00	6.29153E-04	-6.29153E-04	1.76250E+00	-7.58871E-02	2.66667E-02	-1.74134E-03	.00000
160.000	1.04906E+01	4.33056E+00	6.49786E-04	-6.49786E-04	1.76879E+00	-2.67651E-02	2.67799E-02	-1.03573E-03	.00000
170.000	1.03415E+01	4.25588E+00	6.69784E-04	-6.69784E-04	1.76025E+00	1.56443E-02	2.66735E-02	-4.28689E-04	.00000
180.000	1.02029E+01	4.18694E+00	6.89202E-04	-6.89202E-04	1.74296E+00	4.74796E-02	2.64371E-02	2.26512E-05	.00000
190.000	1.00735E+01	4.12301E+00	7.08088E-04	-7.08088E-04	1.72282E+00	6.83701E-02	2.61580E-02	3.12772E-04	.00000
200.000	9.95232E+00	4.06346E+00	7.26483E-04	-7.26483E-04	1.70420E+00	8.02658E-02	2.59010E-02	4.70447E-04	.00000
210.000	9.83842E+00	4.00778E+00	7.44424E-04	-7.44424E-04	1.68953E+00	8.61028E-02	2.57019E-02	5.39032E-04	.00000
220.000	9.73107E+00	3.95555E+00	7.61942E-04	-7.61942E-04	1.67959E+00	8.88124E-02	2.55722E-02	5.61816E-04	.00000
230.000	9.62961E+00	3.90640E+00	7.79066E-04	-7.79066E-04	1.67398E+00	9.07756E-02	2.55062E-02	5.73958E-04	.00000
240.000	9.53351E+00	3.86001E+00	7.95822E-04	-7.95822E-04	1.67167E+00	9.36187E-02	2.54886E-02	5.99466E-04	.00000
250.000	9.44226E+00	3.81612E+00	8.12233E-04	-8.12233E-04	1.67136E+00	9.82192E-02	2.55003E-02	6.51285E-04	.00000
260.000	9.35546E+00	3.77449E+00	8.28318E-04	-8.28318E-04	1.67181E+00	1.04821E-01	2.55226E-02	7.32997E-04	.00000
270.000	9.27272E+00	3.73493E+00	8.44097E-04	-8.44097E-04	1.67198E+00	1.13198E-01	2.55407E-02	8.41251E-04	.00000
280.000	9.19372E+00	3.69725E+00	8.59586E-04	-8.59586E-04	1.67118E+00	1.22835E-01	2.55441E-02	9.68419E-04	.00000
290.000	9.11817E+00	3.66130E+00	8.74802E-04	-8.74802E-04	1.66904E+00	1.33097E-01	2.55276E-02	1.10508E-03	.00000
300.000	9.04580E+00	3.62695E+00	8.89757E-04	-8.89757E-04	1.66551E+00	1.43365E-01	2.54901E-02	1.24209E-03	.00000
310.000	8.97639E+00	3.59406E+00	9.04464E-04	-9.04464E-04	1.66075E+00	1.53133E-01	2.54341E-02	1.37195E-03	.00000
320.000	8.90972E+00	3.56253E+00	9.18937E-04	-9.18937E-04	1.65506E+00	1.62062E-01	2.53644E-02	1.48965E-03	.00000
330.000	8.84561E+00	3.53227E+00	9.33185E-04	-9.33185E-04	1.64888E+00	1.69986E-01	2.52860E-02	1.59271E-03	.00000
340.000	8.78388E+00	3.50317E+00	9.47218E-04	-9.47218E-04	1.64234E+00	1.76889E-01	2.52043E-02	1.68091E-03	.00000
350.000	8.72439E+00	3.47517E+00	9.61047E-04	-9.61047E-04	1.63594E+00	1.82871E-01	2.51232E-02	1.75569E-03	.00000
360.000	8.66699E+00	3.44819E+00	9.74679E-04	-9.74679E-04	1.62979E+00	1.88099E-01	2.50458E-02	1.81951E-03	.00000
370.000	8.61156E+00	3.42217E+00	9.88124E-04	-9.88124E-04	1.62401E+00	1.92765E-01	2.49736E-02	1.87522E-03	.00000
380.000	8.55797E+00	3.39704E+00	1.00139E-03	-1.00139E-03	1.61863E+00	1.97056E-01	2.49072E-02	1.92556E-03	.00000
390.000	8.50612E+00	3.37276E+00	1.01448E-03	-1.01448E-03	1.61361E+00	2.01129E-01	2.48460E-02	1.97287E-03	.00000
400.000	8.45591E+00	3.34927E+00	1.02740E-03	-1.02740E-03	1.60890E+00	2.05103E-01	2.47890E-02	2.01886E-03	.00000
410.000	8.40726E+00	3.32655E+00	1.04017E-03	-1.04017E-03	1.60440E+00	2.09050E-01	2.47352E-02	2.06461E-03	.00000
420.000	8.36007E+00	3.30448E+00	1.05277E-03	-1.05277E-03	1.60004E+00	2.13003E-01	2.46831E-02	2.11062E-03	.00000
430.000	8.31427E+00	3.28311E+00	1.06523E-03	-1.06523E-03	1.59573E+00	2.16965E-01	2.46317E-02	2.15689E-03	.00000
440.000	8.26980E+00	3.26237E+00	1.07755E-03	-1.07755E-03	1.59141E+00	2.20912E-01	2.45803E-02	2.20308E-03	.00000
450.000	8.22657E+00	3.24222E+00	1.08972E-03	-1.08972E-03	1.58706E+00	2.24811E-01	2.45277E-02	2.24870E-03	.00000
460.000	8.18453E+00	3.22265E+00	1.10177E-03	-1.10177E-03	1.58264E+00	2.28622E-01	2.44743E-02	2.29317E-03	.00000
470.000	8.14363E+00	3.20361E+00	1.11368E-03	-1.11368E-03	1.57817E+00	2.32312E-01	2.44203E-02	2.33596E-03	.00000
480.000	8.10381E+00	3.18508E+00	1.12546E-03	-1.12546E-03	1.57366E+00	2.35851E-01	2.43648E-02	2.37665E-03	.00000
490.000	8.06502E+00	3.16705E+00	1.13713E-03	-1.13713E-03	1.56912E+00	2.39221E-01	2.43092E-02	2.41496E-03	.00000

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Subject

RADIAL CRACK

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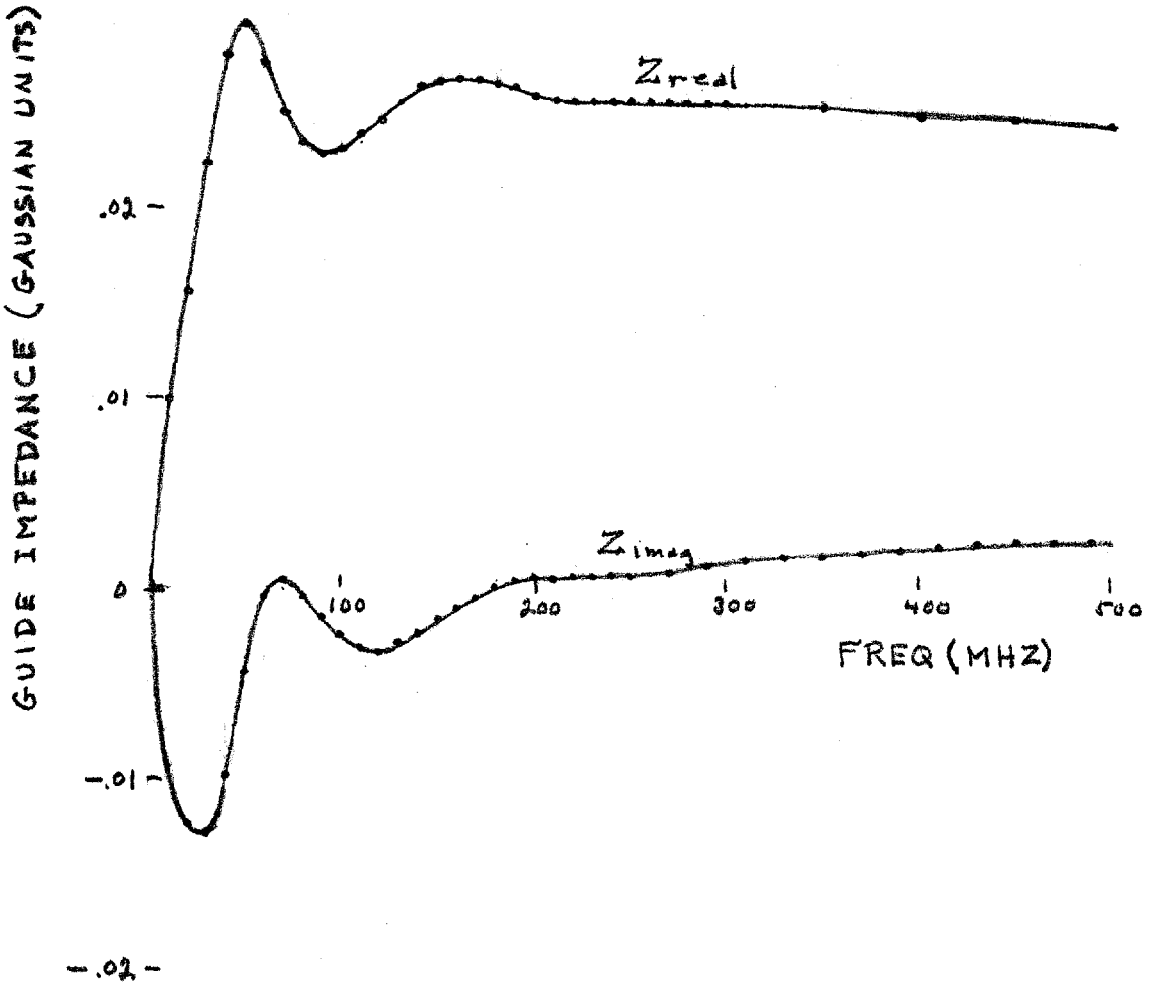
$\delta = .000375$ in
$a = .750$ in
$b = 6.000$ in

$\mu = 100$
$\epsilon = 4.75$

$\sigma_{Fe} = 4.5 \times 10^{12}$ Hz
$\sigma_{Diel} = 9 \times 10^{6}$ Hz